# Entropy methods in combinatorics (L16)

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The entropy $H[X]$ of a discrete random variables $X$ taking values in a set $A$ is defined to be $\sum_{a\in A} p_a \log_2(1/p_a)$, where $p_a$ denotes the probability that $X = a$. For example, if $X$ is uniformly distributed over all $n$-bit 01-strings, then $H[X] = n$. More generally, one can think of $H[X]$ as being something like "the number of bits one typically needs to specify $X$".

This definition, introduced by Claude Shannon in 1948, is fundamental to information theory, coding theory, and important to several other disciplines such as machine learning and biology. Rather more surprisingly, it is a useful tool in extremal combinatorics, and has been at the centre of several recent breakthroughs. It is not that easy to give a unified explanation of just why this is, so the main aim of the course will simply be to demonstrate the use of entropy in all its variety, identifying common themes where possible.

The following topics will approximate the content of the course.

- The axiomatic approach to entropy.
- A special case of Sidorenko's conjecture.
- Brégman's theorem.
- Shearer's lemma, and applications.
- Bounds for the union-closed conjecture.
- Bounds for the sunflower lemma.
- Entropy and sumsets.
- Marton's conjecture for $\mathbf{F}_2^n$

### Prerequisites

This course will start more or less from first principles, but a basic knowledge of extremal graph theory and additive combinatorics will be helpful for setting it in context.

### Literature

There is no single text that covers all the topics above. If you search online for "entropy methods in combinatorics" you will find lecture notes that cover the first few sections. For the rest I recommend searching for material on each topic individually.

### Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.