

Probabilistic Combinatorics (L16)

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This course will introduce a few of the fundamental methods in probabilistic combinatorics. This course will cover some of the following topics.

Basic Probabilistic method

First moment method; deletion method; Lovász local lemma; upper and lower bounds for the Ramsey numbers $R(3, k)$; semi-random method.

Dependent random choice

Extremal numbers of bipartite graphs; Ramsey-Turán; Sidorenko conjecture; Ramsey numbers of hypercubes and bounded degree graphs.

Lower bounds on Ramsey numbers

The Alon–Rödl bound on multi-colour Ramsey numbers; recent progress on $R(3, k)$ and off-diagonal Ramsey numbers.

Graph containers

The Kleitman–Winston containers theorem; discussion of hypergraph containers.

Thresholds for random graphs

Basic examples; threshold for the existence of a Hamilton cycle; spread measures; resolution of the Kahn–Kalai conjecture.

Prerequisites

Familiarity with the material and flavor of Part II Graph Theory (or equivalent) is useful and recommended although not strictly required.

Literature

This course draws from various sources, and there is no one good resource for this course. Some of the main topics are covered in R. Morris and R.I. Oliveira’s lecture notes. Alon and Spencer’s *The probabilistic method* is also a good resource.

1. N. Alon and J. Spencer *The Probabilistic Method*, Wiley (any edition).
2. R. Morris and R.I. Oliveira *Extremal and Probabilistic Combinatorics*, available at https://impa.br/wp-content/uploads/2017/04/28CBM_04.pdf. 2017.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term. Lecture notes will be provided for most of the topics.