Combinatorics (M16)

Prof I.B.Leader

The flavour of the course is similar to that of the Part II Graph Theory course, although we shall not rely on many of the results from that course.

We shall study collections of subsets of a finite set, with special emphasis on size, intersection and containment. There are many very natural and fundamental questions to ask about families of subsets; although many of these remain unsolved, several have been answered using a great variety of elegant techniques.

We shall cover a number of 'classical' extremal theorems, such as those of Erdős-Ko-Rado and Kruskal-Katona, together with more recent results concerning isoperimetric inequalities and intersecting families. There will be several indications of open problems.

We hope to cover the following material.

Set Systems

Definitions. Antichains; Sperner's lemma and related results. Shadows. Compression operators and the Kruskal-Katona theorem. Intersecting families; the Erdős-Ko-Rado theorem.

Isoperimetric Inequalities

Harper's theorem and the edge-isoperimetric inequality in the cube. Inequalities in the grid. The classical isoperimetric inequality on the sphere. The 'concentration of measure' phenomenon. Applications.

Intersecting Families

Katona's t-intersecting theorem. The Ahlswede-Khachatrian theorem. Restricted intersections. The Kahn-Kalai counterexample to Borsuk's conjecture.

Desirable Previous Knowledge

The only prerequisites are the very basic concepts of graph theory.

Introductory Reading

1. Bollobás, B., Combinatorics, C.U.P. 1986.

Additional support

Three examples sheets will be provided and associated examples classes will be given. There will be a one-hour revision class in the Easter Term.