Combinatorics (M16)

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I hope to cover the topics below, in the order indicated. If time remains, which is most unlikely, there are plenty of further exciting topics to present.

Part I.

- Basic Results
- Sperner families and the MBL–Inequality.
- The Erdős–Ko–Rado Theorem.
- The Sauer–Shelah–Perles Inequality.
- The Bollobás Inequality and its extension by Frankl.
- The Kruskal–Katona Inequality and its simplified version, due to Lovász.
- The Box Theorem of Bollobás and Thomason and its applications.
- The Cauchy–Davenport Inequality and its extension, and the Erdős–Ginzburg–Zif Theorem.

Part II.

- The Polynomial Method
- Alon's Combinatorial Nullstellensatz and its applications, including Chevalley's Theorem and the Cauchy-Davenport Theorem.
- The Erdős-Heilbronn Conjecture, proved by the Dias da Silva and Hamidoune. The simple proof of Alon, Nathanson and Ruzsa.
- The conjecture of Kemnitz and Rónyai's theorem.
- The Croot–Lev–Pach lemma and the Ellenberg–Gijswijt theorem.

Pre-requisites

Mathematical maturity and love of combinatorial arguments would be welcome. Familiarity with the Part II Graph Theory course is an asset, but not necessary.

Literature

For the 'classical' results in extremal combinatorics, we shall use my old book below, which was written for Part III courses. This short book is soon to be updated and enlarged with Imre Leader. For the more recent results in the course, most of the original papers are listed below.

1. Alon, N., Combinatorial Nullstellensatz, in *Recent Trends in Combinatorics* (Mátraháza, 1995), *Combin. Probab. Comput.* **8** (1999), 7–29.

2. Bollobás, B., On generalized graphs, *Acta Math. Acad. Sci. Hungar.* **16** (1965) 447–452.

   Bollobás, B., *Combinatorics – Set Systems, Hypergraphs, Families of Vectors and Combinatorial Probability*, Cambridge University Press, Cambridge, 1986. xii+177 pp.

3. Bollobás, B. and A Thomason, Projections of bodies and hereditary properties of hypergraphs, *Bull. London Math. Soc.* **27** (1995) 417–424.

4. Croot, E., V.F. Lev, and P.P. Pach, Progression-free sets in $\mathbf{Z}_n^4$ are exponentially small, *Ann. of Math.* **185** (2017) 331–337.

5. Ellenberg, J.S., and D. Gijswijt, On large subsets of $\mathbf{F}_q^n$ with no three-term arithmetic progression, *Ann. of Math.* (2) **185** (2017) 339–343.

6. Erdős, P., Chao Ko and R. Rado, Intersection theorem for system of finite sets, *Quart. J. Math. Oxford* **12** (1961) 313–318.

7. Frankl, P., An extremal problem for two families of sets, *European J. Combinatorics* **3** 125–127.

8. Lubell, D., A short proof of Sperner's lemma, *J. Combinatorial Theory* **1** (1966), 299.

9. G.O.H. Katona, A Simple Proof of the Erdijs-Chao Ko-Rado Theorem, *Journal of Combinatorial Theory (B)* **13** (1972) 183-184.

10. Sauer, N., On the density of families of sets, *J. Combinatorial Theory Ser. A* **13** (1972) 145–147.

11. Shelah, S., A combinatorial problem; stability and order for models and theories in infinitary languages, *Pacific J. Math.* **41** (1972), 247–261.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a revision class in the Easter Term.