The analysis of Boolean functions (L16)

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Boolean functions, that is, functions from $\{0,1\}^n$ to $\{0,1\}$, are of obvious importance in theoretical computer science, but they also give rise to many important questions in mathematics. One of the major tools for investigating them is Fourier analysis on the group $\mathbf{F}_2^n$, and one of the central concepts in the area is that of the *influence* of a variable on a Boolean function ϕ, which is the probability that changing the value of that variable changes the value of the function. The theory of influences of variables on Boolean functions has numerous applications, not just in theoretical computer science but also in areas as diverse as stability theorems in extremal combinatorics, sharp thresholds in random graphs, and the theory of voting systems. This course will cover the basics of the theory, followed by two central results, the Kahn-Kalai-Linial theorem and Friedgut's junta theorem, after which some of the applications of the theory to those three areas will be presented.

Prerequisites

There are no formal prerequisites other than familiarity with basic definitions in combinatorics and (mainly discrete) probability. Discrete Fourier analysis will be developed from scratch.

Literature

A classic and highly recommended book in the area is Ryan O'Donnell's book Analysis of Boolean Functions, which is available online at https://arxiv.org/abs/2105.10386.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.