Topics in Convex Optimisation (L16)

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Mathematical optimization problems arise in many areas of science and engineering, including statistics, machine learning, robotics, quantum information, and others. The core question of optimization is to compute or approximate a value of the form

$$\min_{x \in X} f(x)$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is an objective function, and $X \subset \mathbb{R}^n$ is a constraint set. Mathematical optimization is a very rich discipline with connections to different areas of mathematics, including convex geometry, numerical analysis, computer science and applied algebraic geometry. In this course, we plan to explore some "modern" topics in convex optimization. A list of tentative topics includes:

- Review of convex analysis and basic convex geometry
- Linear programming, conic programming, duality
- Algorithms and their convergence: (sub)gradient method, proximal gradient method, mirror descent, Nesterov's acceleration, Newton's method
- Semidefinite programming and convex relaxations. The sum-of-squares hierarchy for polynomial optimization problems.

Prerequisites

This course only assumes basic knowledge in linear algebra and analysis. Some knowledge of convex analysis will be useful.

Literature

1. S. Boyd, L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004. http://web.stanford.edu/~boyd/cvxbook/
2. Y. Nesterov, *Introductory Lectures on Convex Optimization*, Springer, 2004. https://doi.org/10.1007/978-1-4419-8853-9
3. S. Bubeck, *Convex Optimization: Algorithms and Complexity*, Foundations and Trends® in Machine Learning, 2014. https://arxiv.org/abs/1405.4980
4. A. Ben-Tal and A. Nemirovski, *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*, SIAM, 2001. http://dx.doi.org/10.1137/1.9780898718829
5. G. Blekherman, P. Parrilo, R. Thomas, *Semidefinite optimization and convex algebraic geometry*, SIAM 2013. http://dx.doi.org/10.1137/1.9781611972290

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.