Mathematical Analysis of the Incompressible Navier-Stokes Equations (M25) – Examinable

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This course will introduce basic mathematical analytical tools for studying nonlinear Parabolic Partial Differential Equations. It will focus the Navier-Stokes equations of viscous incompressible fluids as a prototype of such system of evolution equations. The Navier-Stokes equations appear in a wide range of physical and biological applications, varying from oceanic and atmospheric dynamics, to combustion theory and body-fluid transport. On the one hand, from the mathematical analysis point of view these equations have been recognized to be among the most challenging problems in Applied Analysis. On the other hand, from the computational point of view they are prohibitively expensive to simulate for very large Reynolds numbers, and out of reach even for most powerful state-of-the-art computers.

The course will cover a selection of topics including:

- Introducing the Navier-Stokes and Euler equations of incompressible fluids.
- Sobolev spaces, Sobolev inequalities, interpolation inequalities; and the relevant Functional Spaces.
- Helmholtz Decomposition.
- Steady state solutions to the incompressible Navier-Stokes equations and their regularity.
- Time dependent Leray-Hopf weak solutions to the incompressible Navier-Stokes equations.
- Global regularity of strong solutions for the two-dimensional incompressible Navier-Stokes equations.
- Short time existence of strong solutions in the three-dimensional incompressible Navier-Stokes equations.
- Weak-strong uniqueness and the role of the energy inequality in the Leray-Hopf weak solutions.
- Time analyticity and Gevrey regularity (spatial analyticity) of strong solutions to incompressible Navier-Stokes equations.

Prerequisites

Basic knowledge of Real and Functional Analysis. The rest of the course will be, to a large extend, self-contained.

Literature

1. P. Constantin and C. Foias, *Navier-Stokes Equations*, University of Chicago Press, 1988.
2. J.C. Robinson J.L. Rodrigo and W. Sadowski, *The Three-Dimensional Navier–Stokes Equations : Classical Theory*, Cambridge University Press, 2016.
3. J. C. Robinson, *Infinite-dimensional Dynamical Systems: An Introduction to Dissipative Parabolic PDEs and the Theory of Global Attractors*, Cambridge Texts in Applied Mathematics.

4. R. Temam, *Navier-Stokes Equations: Theory and Numerical Analysis*, North-Holand. New print published by the AMS 2001.

5. R. Temam, *Navier-Stokes Equations and Nonlinear Functional Analysis*, CBMS-NSF Regional Conference Series in Applied Math 66, SIAM, 2nd Ed, 1995.

Additional References

1. A. Chorin and J. Marsden, *A Mathematical Introduction to Fluid Mechanics*, Springer-Verlag.

2. C. Doering and J. Gibbon, *Applied Analysis of the Navier-Stokes Equations*, Cambridge University Press.

3. C. Foias, O. Manley, R. Rosa and R. Temam, *Navier-Stokes Equations and Turbulence*, Cambridge University Press, Cambridge, 2001.

4. O.A. Ladyzhenskaya, The Mathematical Theory of Viscous Incompressible Flow, Mathematics and Its Applications, 2 (Revised Second ed.),(1969) [1963], New York; London; Paris; Montreux; Tokyo; Melbourne: Gordon and Breach.

5. A. Majda and A. Bertozzi, *Vorticity and Incompressible Flow*, Cambridge University Press, 2002.

6. H. Sohr, *The Navier–Stokes Equations, An Elementary Functional Analytic Approach*, Birkhäuser Verlag, Basel, 2001.

7. R. Temam, *Infinite Dimensional Dynamical Systems in Mechanics and Physics*, 2nd Ed, Applied Math Sci. 68, Springer-Verlag, 1997.

Additional support

There will be some examples sheets associated with this course, but some exercise will be assigned during the lectures. Office hours will be by appointment.