Inverse Problems (M24)

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Whenever we have a model for how to take measurements of the state of some underlying object, there is a natural question of whether this state can be recovered from the corresponding measurements. The field of inverse problems concerns itself with exactly this question. Inverse problems arise naturally in many scientific and engineering settings, such as medical imaging (reconstruction problems in magnetic resonance imaging, computed tomography etc.), seismic imaging (e.g. full waveform inversion) and computer vision (e.g. optical flow estimation).

Interesting inverse problems are usually *ill-posed*, meaning that an inverse doesn't exist or, even if a naïve inversion can be defined, its output is unstable, making it useless in applications: measurements are generally corrupted by noise, which will be blown up by the inversion.

In this course, we will study methods that can be used to overcome this ill-posedness, with a focus on applications to imaging. The course will consist of two main parts, the first of which studies deterministic methods, through the classical *regularisation theory* of (linear) inverse problems. In this part, we will pay special attention to so-called *variational regularisation* methods, which can be efficiently solved using modern (convex) optimisation methods. In the second part, we will take a *Bayesian* statistical perspective on inverse problems, which is a natural framework for studying stochastic methods for inverse problems.

Prerequisites

This course assumes basic knowledge of linear algebra, analysis (as treated in Part II Linear Analysis or Analysis of Functions) and probability theory (as treated in Part II Probability and Measure). Additional knowledge in convex analysis and Bayesian statistics is beneficial, but not strictly necessary.

Literature

1. H. W. Engl, M. Hanke and A. Neubauer. *Regularization of Inverse Problems.* Vol. 375, Springer Science & Business Media, 1996, ISBN: 9780792341574.

2. O. Scherzer, M. Grasmair, H. Grossauer, M. Haltmeier and F. Lenzen. *Variational Methods in Imaging.* Applied Mathematical Sciences, Springer New York, 2008, ISBN: 9780387309316.

3. M. Dashti and A. M. Stuart. *The Bayesian Approach to Inverse Problems,* in: *Handbook of Uncertainty Quantification.* Springer Cham, 2017, ISBN: 9783319123844.

4. A. M. Stuart, *Inverse problems: a Bayesian perspective.* Acta Numerica, **19**, pp. 451–559, 2010. https://doi.org/10.1017/S0962492910000061.

5. M. Benning, M. Burger, *Modern Regularization Methods for Inverse Problems.* Acta Numerica, **27**, pp. 1—111, 2018. https://doi.org/10.1017/S0962492918000016.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.