Distribution Theory and Applications (L16)

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This course will give an introduction to the theory of distributions and its application to the study of linear PDEs. We aim to make mathematical sense of objects like the Dirac delta function and find out how to meaningfully take the Fourier transform of a polynomial. The course will focus on the *use* of distributions, rather than the functional-analytic foundations of the theory.

First we will cover the basic definitions for distributions and related spaces of test functions. Then we look at operations such as differentiation, translation, convolution and the Fourier transform. We will introduce the Sobolev spaces $H^s(\mathbf{R}^n)$ and $H^s_{\mathrm{loc}}(X)$ and describe them in terms of Fourier transforms for tempered distributions. The material that follows will address questions such as

- What does a generic distribution look like?
- Why are solutions to Laplace's equation always infinitely differentiable?
- Which functions are the Fourier transform of a distribution with compact support?

i.e. structure theorems, elliptic regularity, Paley-Wiener-Schwartz. The final section of the course will be concerned with Hörmander's theory of oscillatory integrals.

Prerequisites

Elementary concepts from undergraduate real analysis. Some knowledge of complex analysis would be advantageous (e.g. the level of IB complex methods or analysis). Knowledge of measure theory and functional analysis will *not* be needed, but might be complementary.

Preliminary Reading

1. F.G. Friedlander & M.S. Joshi, *Introduction to the Theory of Distributions*, C.U.P, 1998.
2. M. J. Lighthill, *Introduction to Fourier Analysis and Generalised Functions*, C.U.P, 1958.
3. G.B. Folland, *Introduction to Partial Differential Equations*, Princeton Univ Pr, 1995.

Literature

1. L. Hörmander, *The Analysis of Linear Partial Differential Operators: Vols* I-II, Springer, 1985.
2. M. Reed & B. Simon, *Methods of Modern Mathematical Physics: Vols* I-II, Academic Press, 1979.
3. F. Trèves, *Linear Partial Differential Equations with Constant Coefficients*, Routledge, 1966.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. Model solutions will be made available. There will be a revision class in the Easter Term.