Approximation Theory (M16)

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The course will give an overview of basic concepts of Approximation Theory, i.e., best and good approximation of a large family of functions by a smaller set (usually finitely generated, linear or nonlinear) in certain normed spaces (such as C and L_p), construction of good approximants (in various settings), finding approximation order for smooth functions (say, order n^{-k} for approximation by polynomials of degree n).

The course consists of three parts.

- We start with the classical approximation by polynomials, which includes the Weierstrass theorem, positive linear operators, and direct and inverse theorems for trigonometric approximation.
- We move then the emphasis to univariate splines which are piecewise polynomial functions. Here we study representation through the B-spline basis, spline interpolation theory and norm-minimization property of splines via orthogonal spline projector.
- Finally, we make a tour into wavelets which will cover the multiresolution analysis and Daubechies orthogonal wavelets with a compact support.

Prerequisites

The course is mostly self-contained, but it assumes a standard mathematical analysis background (say, normed linear spaces, inner products, Fourier series) and some linear algebra (solution of linear systems of equations). Some functional analysis tools (e.g., Hahn-Banach theorem) appear in comments and advanced problems, so they are helpful but not required.

Literature

To a large extent, the course follows the Lecture Notes by C. de Boor where one can find much more details on each topic. In the first part, it is also based on Cheney's book.

1. C. de Boor, Notes on Approximation Theory
   https://pages.cs.wisc.edu/~deboor/ma887.html
2. E. W. Cheney, *Approximation theory*, MgGrow-Hill, New-York, 1966.
3. R. A. DeVore, G. G. Lorents, *Constructive Approximation,* Springer-Verlag, Berlin, 1993.
4. I. Daubechies, Ten lectures on wavelets, CBMS-NSF Regional Conference Series in Applied Mathematics, 61, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1992.

Additional suppport

Three examples sheets will be provided and three associated examples classes will be given. There will be a revision class in the Easter term.