

Applied Analysis (M24)

George Stepaniants and David Stuart

Scientific applications frequently involve the analysis of differential and integral operators, which requires as a first step to interpret them as operators on specific function spaces, typically complete normed spaces or more general topological vector spaces. We will develop the theory of operators on such spaces, and then discuss techniques such as implicit function theorems, Calculus of Variations and Bifurcation theory and their uses for nonlinear problems.

Part I Foundational Material

- (a) Metric spaces: examples of Hilbert and Banach spaces. Integration theory. Distributions and Schwartz space. (Review).
- (b) Integral and differential operators as unbounded operators on normed spaces. Integral equations. Spectral theorem for self-adjoint operators and applications.

Part II Calculus of Variations: the direct method. Lower semi-continuity and convexity. Weak convergence, Gamma convergence and Young Measures. Applications in physics, material science, image processing...

Part III Implicit function theorems in Banach space and applications.

- (a) Bifurcation theory and Lyapunov Schmidt reduction.
- (b) Monotone operators. Gradient flows. Applications to nonlinear problems.

Prerequisites

The part II courses *Linear Analysis*, *Analysis of Functions* and *Probability and Measure* are most relevant, and the crucial topics in them will be reviewed; either familiarity, or a willingness to refer to the notes as needed, is expected. Applications will be introduced in a self-contained way, with the aim to illustrate the application of general analytical principles and methods, with no particular scientific background assumed.

Literature

- A Ambrosetti and G Prodi, *A Primer of Nonlinear Analysis*. CUP, 1993.
- H Brezis *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer
- M Schechter, *An Introduction to Nonlinear Analysis* CUP, 2010.
- M Reed and B Simon, *Methods of Mathematical Physics I: Functional Analysis*, Academic press, 1981.
- J Hunter and B Nachtergaele, https://www.math.ucdavis.edu/~bxn/applied_analysis.pdf

Additional support

Weekly office hour. Four examples sheets will be provided and associated examples classes will be given. There will be a one-hour revision class in the Easter Term.