Functional Analysis (M24)

Dr A Zsák

This course covers many of the major theorems of abstract Functional Analysis. It is intended to provide a foundation for several areas of pure and applied mathematics. We will cover as many of the following topics as time permits.

- Hahn–Banach theorems on the extension of linear functionals. Locally convex spaces. Discussion of the duals of the spaces $L_p(\mu)$ and $C(K)$.
- Weak and weak-star topologies. Hahn–Banach theorems on separation of convex sets. Theorems of Mazur, Goldstine and Banach–Alaoglu. Reflexivity and local reflexivity. Uniform convexity. The Milman–Pettis theorem. Duals of the spaces $L_p(\mu)$.
- Extreme points and the Krein–Milman theorem. Partial converse and the Banach–Stone theorem.
- Schauder bases in Banach spaces. Basis techniques and applications: compact and strictly singular operators; Pełczyński's decomposition method; the Eberlein–Šmulian theorem.
- The Bochner integral. The Krein–Šmulian theorem.
- Non-linear theory of Banach spaces. Lipschitz functions and the Lipschitz classification problem. Spaces with the Radon–Nikodym property. Linearization via differentiability and Rademacher's theorem. Linearization via invariant means. Uniform classification and Ribe's theorem.
- Banach algebras, elementary spectral theory. Commutative Banach algebras and the Gelfand representation theorem. Holomorphic functional calculus.
- Hilbert space operators, C^*-algebras. The Gelfand–Naimark theorem. Spectral theorem for commutative C^*-algebras. Spectral theorem and Borel functional calculus for normal operators.

Prerequisites

Thorough grounding in basic topology and analysis. Some knowledge of basic functional analysis and basic measure theory. In spectral theory we will make use of basic complex analysis. For example, Cauchy's theorem, Cauchy's integral formula and the maximum modulus principle. Much of the background material will be recalled either in lectures or via handouts.

Literature

1. Albiac, Fernando and Kalton, Nigel J. *Topics in Banach space theory*, Springer, 2016.
2. Allan, Graham R. *Introduction to Banach spaces and algebras (prepared for publication by H. Garth Dales).* Oxford University Press, 2011.
3. Bollobás, Béla *Linear analysis: an introductory course.* Cambridge University Press, 1990.
4. Diestel, Joseph *Sequences and series in Banach spaces*, Springer, 1984.
5. Lindenstrauss, Joram and Tzafriri, Lior *Classical Banach spaces. I*, Springer, 1977.

6. Murphy, Gerard J. *C^*-Algebras and Operator Theory.* Academic Press, Inc., 1990.

7. Rudin, Walter *Real & Complex Analysis.* McGraw-Hill, 1987.

8. Rudin, Walter *Functional Analysis.* McGraw-Hill, 1990.

9. Taylor, S. James *Introduction to measure and integration.* Cambridge University Press 1973.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term. There will be some material as well as examples sheets and announcements available at `www.dpmms.cam.ac.uk/~az10000/`