Analysis of Partial Differential Equations (M24)

Dr Z. Wyatt

This course serves as an introduction to the mathematical study of Partial Differential Equations (PDEs). The theory of PDEs is presently a major area of active research, and it goes back to the very birth of mathematical analysis in the 18th and 19th centuries. The subject lies at the crossroads of physics and many areas of pure and applied mathematics. The course will mostly focus on modern techniques, with emphasis on functional analytic techniques, relying on *a priori* estimates rather than explicit solutions. The course will primarily focus on approaches to linear, elliptic and evolutionary problems through weak formulations and energy estimates, with the prototypical examples being Laplace's equation, the heat equation, the transport equation, the wave equation and Schrödinger's equation. The following topics will be studied.

- *Introduction and Cauchy–Kovalevskaya theorem.* Definition and examples of PDEs; well-posed problems; classification; real analyticity; Cauchy–Kovalevskaya theorem.
- *Spaces of functions.* Hölder spaces; weak derivatives; Sobolev spaces; approximation by mollifiers; extension and trace theorems; Sobolev embeddings.
- *Elliptic boundary value problems.* Weak formulation; solvability via Lax–Milgram; Fredholm alternative for elliptic PDEs and its consequences on the spectrum; regularity of solutions via the difference quotient method.
- *Hyperbolic equations.* Notion of hyperbolicity for second-order linear operators; weak formulation of the initial boundary value problem; existence via Galerkin's method; uniqueness via the vector field method; finite speed of propagation; first-order hyperbolic PDEs and the method of characteristics.

Prerequisites

There are no specific pre-requisites beyond a standard undergraduate analysis background, in particular a familiarity with measure theory and integration. The course will be mostly self-contained and can be used as a first introductory course in PDEs for students wishing to continue with some specialised PDE Part III courses in the Lent and Easter terms.

Preliminary Reading

Students are strongly encouraged to read through Claude Warnick's notes for the Part II Analysis of Functions course https://www.dpmms.cam.ac.uk/~cmw50/Teaching/Analysis-of-PDE/page1.html. Some lecture notes from previous iterations of the course are available online at http://cmouhot.wordpress.com/teachings/ and https://www.dpmms.cam.ac.uk/~zw253/teaching.html.

Literature

1. Evans, L. C., *Partial Differential Equations*, Springer, 2010
2. Brezis, H., *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer,2010.
3. John, F., *Partial Differential Equations*, Springer, 1991

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.