Elliptic Partial Differential Equations (L24)

Dr. G. Orriols

This course will provide an introduction to the theory of linear second order elliptic partial differential equations. Second order elliptic equations play a fundamental role in many areas of mathematics including Calculus of Variations, Riemannian Geometry and Mathematical Physics. Linear elliptic theory provides the foundation for studying a number of non-linear problems arising in these fields, such as minimal submanifolds, harmonic maps, special metrics, and evolution problems in Geometry and Mathematical Physics.

We will develop a rigorous treatment of both classical and weak solutions to linear elliptic equations, focusing on the questions of existence and uniqueness of solutions to the Dirichlet problem and regularity of solutions. Specific topics include harmonic functions, maximum principles for general second order equations, Schauder estimates, the continuity method for existence of solutions, Perron's method, divergence form operators, and De Giorgi–Nash–Moser estimates, with the general goal of explaining the solution of Hilbert's XIX problem concerning regularity of minimizers of variational integrals.

Prerequisites

Part III *Analysis of PDEs.*

Literature

1. D. Gilbarg and N. Trudinger, *Elliptic partial differential equations of second order.*
2. M. Giaquinta, L. Martinazzi, *An Introduction to the Regularity Theory for Elliptic Systems, Harmonic Maps and Minimal Graphs*

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.