Analysis of Partial Differential Equations (M24)

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This course serves as an introduction to the mathematical study of Partial Differential Equations (PDEs). The theory of PDEs is presently a huge area of active research, and it goes back to the very birth of mathematical analysis in the 18th and 19th centuries. The subject lies at the crossroads of physics and many areas of pure and applied mathematics.

The course will mostly focus on developing the theory and methods of the modern approach to PDE theory. Emphasis will be given to functional analytic techniques, relying on a priori estimates rather than explicit solutions. The course will primarily focus on approaches to linear, elliptic and evolutionary problems through energy estimates, with the prototypical examples being Laplace's equation, the heat equation, the wave equation and Schrödinger's equation.

The following concepts will be studied:

- well-posedness
- the Cauchy problem for general (non-linear) PDE
- characteristics
- Sobolev spaces
- elliptic boundary value problems (solvability and regularity)
- evolutionary problems (hyperbolic, parabolic and dispersive PDE)

Prerequisites

There are no specific prerequisites beyond a standard undergraduate analysis background, in particular a familiarity with measure theory and integration. The course will be mostly self-contained and can be used as a first introductory course in PDEs for students wishing to continue with some specialised PDE Part III courses in the Lent and Easter terms.

Preliminary Reading

Students are strongly encouraged to read through Claude Warnick's notes for the Part II Analysis of Functions course. These will be made available online. The following article also gives an overview of the field of PDEs

1. Klainerman, S., *Partial Differential Equations*, Princeton Companion to Mathematics (editor T. Gowers), Princeton University Press, 2008.

Literature

1. Some lecture notes from a previous lecturer of the course are available online at:

   http://cmouhot.wordpress.com/teachings/

   The following textbooks are excellent references:

2. Evans, L. C., *Partial Differential Equations*, Springer, 2010.

3. Brezis, H., *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer,2010.

4. John, F., *Partial Differential Equations*, Springer, 1991.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term. There will be one office hour a week.