Introduction to motivic cohomology (L16)

Non-Examinable (Graduate Level)

Professor M. Gross

The main goal of this course is to introduce Voevodsky's theory of motivic cohomology of smooth schemes as expositied in reference 1. In the large, motivic cohomology is meant to be a "universal cohomology theory" for schemes which specializes, in various settings, to better-known cohomology theories such as étale cohomology.

Voevodsky constructs his cohomology groups as the cohomology of certain Zariski sheaves $\mathbf{Z}(q)$ for q a non-negative nteger, and these sheaves are defined using algebraic (Chow) cycles. In fact, the cohomology of these sheaves coincides with the previously constructed higher Chow groups of Bloch (see reference 3), but Voevodsky's construction enables many new results.

I will first present the construction of Bloch's higher Chow groups, as this is quite elementary and will rely mainly on material covered in the first several weeks of the course on Intersection Theory. I will then cover Voevodsky's construction, and depending on time remaining, will consider a selection of possible topics.

Prerequisites

Having a background in Algebraic Geometry at least to the level of the Part III course is crucial. Familiarity with Chow groups will also be essential. As this will be covered in the Lent course on Intersection Theory, it should be possible to follow this course while following the Intersection Theory course simultaneously.

Literature

1. Mazza, Carlo; Voevodsky, Vladimir; Weibel, Charles. *Lecture notes on motivic cohomology.* Clay Mathematics Monographs, 2. American Mathematical Society, Providence, RI; Clay Mathematics Institute, Cambridge, MA, 2006. xiv+216 pp.

2. Voevodsky, Vladimir; Suslin, Andrei; Friedlander, Eric M. *Cycles, transfers, and motivic homology theories.* Annals of Mathematics Studies, 143. Princeton University Press, Princeton, NJ, 2000. vi+254 pp.

3. Bloch, Spencer. *Algebraic cycles and higher K-theory.* Adv. in Math. 61 (1986), no. 3, 267–304.