Algebraic Geometry (M24)

Dr. S. Canning

This course is an introduction to modern algebraic geometry. We develop the basic theory of sheaves, schemes, and morphisms thereof. After studying properties of morphisms of schemes, we will study (quasi)coherent sheaves, with a particular focus on line bundles, Cartier divisors, and the Picard group. Throughout the course, there will be an emphasis on the geometric aspects of the theory. Near the end of the course, we will give an introduction to sheaf cohomology. These topics comprise part of the basic toolkit of algebraic geometry, and are of fundamental importance for every algebraic geometer. They are also useful for those in nearby fields, including but not limited to differential geometry, number theory, and topology.

Prerequisites

The basic theory of rings, modules, and elementary topology will be assumed. Intuition from complex analysis and algebraic and differential topology will be helpful, but not formally necessary. The contents of Part II Algebraic Geometry (the basic geometry of affine and projective varieties) is also useful but not formally necessary. Students are recommended to have taken a previous course in Commutative Algebra or to take one in parallel with Algebraic Geometry. The most important content is in Chapters 1–3 and 5–7 of *Introduction to Commutative Algebra* by Atiyah and Macdonald.

Literature

Both of the texts below cover similar material, but are stylistically quite different. It is recommended to peruse both to find which one suits you best.

1. R. Hartshorne *Algebraic Geometry.* Springer, 1977. https://link.springer.com/book/10.1007/978-1-4757-3849-0.
2. R. Vakil, *The Rising Sea. Foundations of Algebraic Geometry.* Princeton University Press, 2025. https://math.stanford.edu/~vakil/216blog/FOAGoct2125public.pdf.

Additional support

There will be four examples sheets and four associated examples classes. There will be a one-hour revision class in the Easter Term.