Abelian Varieties (L24)

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An abelian variety is an irreducible projective variety over a field which is also a group. The simplest example is an elliptic curve. Other examples include the Jacobian variety of a (smooth projective) curve, whose points parametrise divisor classes of degree zero. There are beautiful analytic and algebraic theories of such objects. They also crop up in other areas of geometry and in number theory.

Although elliptic curves can be studied using fairly elementary algebraic geometry and explicit equations, for abelian varieties of higher dimension scheme theory is pretty much essential, and gives a much better understanding (as well as a good illustration of how powerful scheme theory is). This course will therefore be in 2 parts.

The first 3–4 weeks will be a continuation of the Algebraic Geometry course from Michaelmas. We will cover various topics in abstract algebraic geometry. One important topic will be the study of how cohomology varies in a family (cohomology and basechange).

In the remainder of the lectures, we will introduce group schemes, and develop some of the algebraic theory of abelian varieties, approximately covering Chapter 2 of Mumford's book. If time permits, we may also give the construction of the Jacobian of a curve.

Prerequisites

Algebraic Geometry (MT) and Commutative Algebra (MT) or equivalent are essential.

Literature

- 1. David Mumford: *Abelian varieties*. Second edition. Tata Institute of Fundamental Research, 1974.
- 2. Edixhoven, Moonen and Van Der Geer: draft book, available at http://van-der-geer. nl/~gerard/AV.pdf or https://www.math.ru.nl/~bmoonen/research.html#bookabvar
- 3. Robin Harsthorne: *Algebraic geometry*. Springer, corrected edition. (For the first half of the course)

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term.