

Abelian Varieties (L24)

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An abelian variety is an irreducible projective variety over a field which is also a group. The simplest example is an elliptic curve. Other examples include the Jacobian variety of a (smooth projective) curve, whose points parametrise divisor classes of degree zero. There are beautiful analytic and algebraic theories of such objects. They also crop up in other areas of geometry and in number theory. This course will cover some of the basic algebraic theory of abelian varieties. Topics to be covered may include:

- Abelian varieties, line bundles and polarisations.
- The Jacobian of a curve.
- Abelian varieties over finite fields. The Weil conjectures.

Although elliptic curves can be studied using fairly elementary algebraic geometry, for abelian varieties of higher dimension scheme theory is pretty much essential, and gives a much better understanding (as well as a good illustration of how powerful scheme theory is). So the first 6 or lectures will be devoted to some results and concepts from algebraic geometry not covered in the Michaelmas Term course: differentials, smooth and étale morphisms, flatness, more about cohomology of coherent sheaves.

Prerequisites

Algebraic Geometry (MT) or equivalent is essential.

Literature

1. David Mumford: *Curves and their Jacobians*. University of Michigan Press, 1977.
2. David Mumford: *Abelian varieties*. Second edition. Tata Institute of Fundamental Research, 1974.
3. Edixhoven, Moonen and Van Der Geer: draft book, available at <http://van-der-geer.nl/~gerard/AV.pdf> or <https://www.math.ru.nl/~bmoonen/research.html#bookabvar>

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.