

Recent progress on Hilbert’s sixth problem (L16)

Non-Examinable (Part III Level)

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One part of Hilbert’s sixth problem, presented at the second International Congress of Mathematicians (Paris, 1900), is “*the problem of developing mathematically the limiting processes [...] which lead from the atomistic view to the laws of motion of continua*”. In modern terms: how do kinetic and fluid equations emerge from Newtonian mechanics, and how does irreversible behaviour arise from reversible microscopic dynamics? Boltzmann’s molecular chaos hypothesis provided a conceptual blueprint, but controlling the correlations and memory effects created by recollisions long seemed mathematically out of reach. A first breakthrough was Lanford’s 1975 theorem, which rigorously justifies the limit from Newton’s equations to the Boltzmann equation for short times. Yu Deng was awarded a Fields Medal in 2026, in part for his work with Zaher Hani and Xiao Ma completing this programme.

This course centres on their papers extending Lanford’s theorem to long times and deriving the compressible Euler and incompressible Navier–Stokes–Fourier equations from hard-sphere Newtonian dynamics. Together these provide the first complete derivation along the Newton–Boltzmann–fluid route for a dilute hard-sphere gas, resolving this part of Hilbert’s sixth problem. The long-time result holds on arbitrarily long time intervals, as long as the limit solution remains sufficiently regular, provided the number of particles is large enough (and the Knudsen number is small enough for the fluid limits). The emphasis will be on the structure of the proofs and selected key estimates. If time permits, we will contrast these results with recent progress on singular mean-field limits: the derivation of the two-dimensional Vlasov–Poisson system from particles with Coulomb interactions without a cutoff, and the three-dimensional problem, which remains open without a cutoff.

Pre-requisites. Probability and measure theory, analysis of functions, basic PDE theory. Some familiarity with kinetic theory or fluid mechanics would be helpful but is not required: the necessary background will be introduced in the course.

Literature

1. I. Gallagher, L. Saint-Raymond & B. Texier, *From Newton to Boltzmann: Hard Spheres and Short-range Potentials*. Zurich Lectures in Advanced Mathematics, EMS, 2013. [arXiv:1208.5753](#).
2. Y. Deng, Z. Hani & X. Ma, *Long time derivation of the Boltzmann equation from hard sphere dynamics*. Preprint, 2024, revised 2025. [arXiv:2408.07818](#).
3. Y. Deng, Z. Hani & X. Ma, *Hilbert’s sixth problem: derivation of fluid equations via Boltzmann’s kinetic theory*. Preprint, 2025. [arXiv:2503.01800](#).
4. M. Duerinckx & P.-E. Jabin, *Derivation of 2D Vlasov–Poisson for classical particles with Coulomb interactions*. Preprint, August 2026. [arXiv:2608.04104](#).
5. M. Feistl-Held & P. Pickl, *On the mean-field limit for the Vlasov–Poisson system*. Journal of Statistical Physics **193** (2026), article 129. [arXiv:2504.01471](#).