Algebraic Geometry (M24)

Professor Mark Gross

The course will be an introduction to modern algebraic geometry. The first part of the course will develop the basic theory of sheaves, and use this to give a treatment of schemes, morphisms between them, and fibre products. We will then explore morphisms the basic properties of morphisms, including separatedness and properness. After this, we turn to the study of quasi-coherent sheaves, with a particular focus on line bundles, the Picard group, and other proximate notions. The final few lectures in the course will give a rapid introduction to sheaf cohomology "for the working mathematician".

The course will develop the basic toolkit used by practicing algebraic geometers, and will also be appropriate for anyone with an interest in acquiring a working knowledge of algebraic geometry, including those in nearby fields such as differential geometry, topology, and number theory.

Prerequisites

The basic theory of rings and modules, and elementary topology will all be assumed. It is recommended that students have had a previous course in Commutative Algebra, take one in parallel, or familiarize themselves with the concepts from Atiyah–MacDonald's text on the subject. In the latter case, the crucial content is in Chapters 1–3 and 5–7. The key ideas from algebra that we will need are ideals, modules, tensor products, and localization. Basic intuition is provided from topology and the theory of manifolds, and this may be helpful but is not essential.

A prior course in the algebraic geometry of affine and projective varieties is not logically necessary for the course, i.e. the contents of the Part II Algebraic Geometry course. However, students should be aware that certain key examples in scheme theory do come from classical algebraic geometry. Students with no prior exposure to algebraic geometry may occasionally need to look up such constructions. However, this will be the exception rather than the rule, and ample references will be provided.

Literature

The following books complement the course material. The first and second are roughly interchangeable. The final text provides valuable intuition for the many unfamiliar concepts in scheme theory.

1. R. Hartshorne, *Algebraic Geometry*, Springer (1977).
2. R. Vakil, *The rising sea. Foundations of algebraic geometry.* Available online.
3. D. Eisenbud and J. Harris, *Geometry of schemes*, Springer (2000).

Additional support

Four examples sheets will be provided and four associated examples classes will be given; these will be essential. Additional problems and examples may be provided to clarify more difficult concepts. There will be a revision class in the Easter Term.