

Algebraic Geometry II: Curves and Surfaces (L24)

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This will be a follow-up to the Michaelmas Term Part III course in algebraic geometry. The goal of the course is to use the basic theory developed there – schemes, sheaves, and cohomology – on concrete examples and gain facility with actual calculations in the subject. We will focus on the geometry of projective varieties of low dimension, and especially on the basic theory of curves and surfaces.

The course will include the following:

- A review of sheaf cohomology and the definition of basic invariants of curves and surfaces, such as the genus, irregularity, and Kodaira dimension.
- Geometric manipulations of algebraic varieties – projective bundles, blowups, branched covers.
- Classification of curves of small genus.
- Construction of algebraic surfaces with specified invariants, and many examples, including K3 surfaces, Enriques surfaces, and elliptic fibrations.
- Geometry of del Pezzo surfaces (including a discussion of the 27 lines on the cubic surface).
- Birational geometry of surfaces; the existence of minimal birational models and Castelnuovo's contraction theorem.

Prerequisites

The essential prerequisite is Part III Algebraic Geometry. Familiarity with some aspects of classical algebraic geometry, for example as covered in the Part II Algebraic Geometry course, will be useful but not essential. This course assumes you know nothing but can do anything.

Literature

1. R. Hartshorne, *Algebraic Geometry*. Springer-Verlag, 1977.
2. A. Beauville, *Complex Algebraic Surfaces*. 2nd edition. Cambridge University Press, 1996.

Additional support

Four example sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.