Intersection Theory (L24)

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As every mathematician knows, counting is hard. In algebraic geometry, an important class of questions is how many geometric objects of a particular specified type there are that satisfy certain constraints. For example: How many smooth plane conics are tangent to five generic plane conics? How many twisted cubic curves are tangent to 12 quadrics? The answers to these questions (3264 and 5,819,539,783,680, respectively) are interesting, but not nearly so interesting as the fact that such questions *have* answers, when correctly posed.

Intersection theory is the language and machinery in algebraic geometry, analogous to that of cohomology for smooth manifolds, that is used to rigorously answer counting questions. It is a central part of the subject; its study allows one to get to know many of algebraic geometry's most important objects, and its modern branches, such as Gromov-Witten Theory, are themselves massive areas of ongoing research. As such, this course would serve well as a second (or perhaps third) course in algebraic geometry, following from Part III Algebraic Geometry.

We will develop the basic theory of Chow groups, including cycles, rational equivalence, flat pullbacks, Chern classes, Segre classes, and intersection products. We will then use these tools to compute a few important examples, such as the Chow ring of a Grassmannian. Finally, we will answer a few counting questions: *e.g.* lines on a smooth cubic surface, conics through 8 lines in projective 3-space, &c.

Our primary texts will be [1], which is the definitive exposition of this subject, and [2], which is an excellent beach read.

Prerequisites

An understanding of algebraic varieties, schemes, and sheaves as covered in Part III Algebraic Geometry. A familiarity with some algebraic topology (especially singular cohomology and Poincaré duality) will help, but is not strictly required.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

References

1. W. Fulton, *Intersection Theory*, Springer, 1998. doi: 10.1007/978-1-4612-1700-8
2. David Eisenbud and Joe Harris, *3264 and All That: A Second Course in Algebraic Geometry*, Cambridge University Press, 2016. ISBN: 9781107017085