Lie algebras and their Representations (M24)

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This course is an introduction to the properties and representations of semisimple complex Lie algebras. The structure and representation theory of semisimple Lie algebras is one of the most beautiful and wide-reaching subjects in mathematics. It has applications to number theory, topology, algebraic geometry, and theoretical physics, to name a few examples. Lie algebras arise as tangent spaces to certain manifolds called Lie groups, yet they can be defined purely algebraically, which simplifies their study. The representation theory of a complex simple Lie algebra can be understood in terms of combinatorial data (e.g. roots, weights, and Weyl groups). Understanding this data will be at the heart of this course.

The following is an outline of the topics that will be covered in the course:

1. The basics of (general) Lie algebras and their representations.

2. Representation theory of sl_2.

3. Root systems, Weyl groups, and the classification of simple complex Lie algebras.

4. The classification of finite-dimensional representations of simple complex Lie algebras, Verma modules, and the Weyl character formula.

Prerequisites

The only real prerequisite is linear algebra. It will be beneficial, but not strictly necessary, to have some familiarity with group actions, modules and basic representation theory, as well as tensor products, symmetric powers, and exterior powers of vector spaces.

Literature

1. Humphreys, James E. *Introduction to Lie algebras and representation theory*, Graduate Texts in Mathematics, 9, Springer-Verlag, New York-Berlin, 1978. (The canonical reference, which is closest to the route we will take.)

2. Fulton, William and Harris, Joe. *Representation theory*, Graduate Texts in Mathematics, 129, Springer-Verlag, New York, 1991. (Contains a wealth of examples. Lecture 1 is a good introduction to representation theory of finite groups that motivates some of the results that we will discuss.)

3. Carter, Roger W. *Simple groups of Lie type, Wiley Classics Library*, John Wiley & Sons, Inc., New York, 1989.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.