Coxeter Groups

Dr A. Jones

This course will examine the theory of Coxeter groups from an algebraic perspective, focusing on their applications within the representation theory of algebraic groups.

Coxeter groups are abstractly defined as groups generated by *involutions*, i.e. elements of order 2. As such, they arise very naturally throughout geometry, as reflectional symmetry groups. In algebra, they also play a major role in Lie theory, in the form of the Weyl group of a semisimple Lie algebra or a reductive Lie group, structures of fundamental importance within modern number theory and physics.

We will cover the following topics, building up the theory of Coxeter groups from their most basic examples, exploring their key properties and some applications.

- Finite and linear reflection groups.
- Definition and basic properties of Coxeter systems, Coxeter diagrams, length functions and irreducibility.
- Geometric representation, and classification of finite Coxeter groups.
- Affine, Spherical, Euclidean and Hyperbolic groups, examples from Lie theory.
- Groups with (B, N)-pairs and buildings.
- Introduction to Hecke algebras and Kazhdan-Lustig theory.

Prerequisites

Part IA Geometry, Part IA Groups and Part IB Linear Algebra are definite prerequisites. An understanding of representation theory is desirable, so Part IB Groups, Rings and Modules and Part II Representation Theory are recommended, as is very strongly the additional Part III course Lie algebras and their Representations.

Literature

Our main resource will be Humphreys' book [4], but the other references below also give a good overview of the theory, and some further reading, including applications in Lie theory [2], Kazhdan-Luztig theory [3], and the theory of buildings [1,6] that we will not have time to explore in detail. See [5] for a more geometric approach to the subject.

1. K. S. Brown, *Buildings,* Springer-Verlag, New York (1989).
2. R. W. Carter, *Simple Groups of Lie type,* Wiley Classics Library, John Wiley & Sons, Inc., New York, (1989 reprint).
3. B. Elias; S. Makisumi; U. Thiel; G. Williamson, *Introduction to Soergel Bimodules,* RSME Springer Series Volume 5 (2020).
4. J. E. Humphreys, *Reflection groups and Coxeter groups,* Cambridge University Press, Cambridge (1990).

5. A. Thomas, *Geometric and Topological aspects of Coxeter groups and Buildings,* European Mathematical Society (2018).

6. M. Ronan, *Lectures on Buildings,* Perspectives in Mathematics **7** (1989).

Additional support

There will be 24 one-hour lectures given in Lent term 2026, as well as four examples classes. Four examples sheets will be provided, to be discussed during the examples classes. There will also be a revision class in Easter Term 2026 before the exam.