

Commutative Algebra (M24)

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This course will provide an introduction to the theory of commutative rings and modules over these rings. It is a foundational course for algebraic geometry and algebraic number theory, the areas it takes its motivation from. Commutative rings naturally appear in these respective subjects as coordinate rings of affine varieties and rings of algebraic integers of number fields.

Though these rings may seem very different at first, viewed through the more abstract lens of commutative algebra, they become much more similar. One obvious point of contact is the importance of prime ideals, corresponding to prime numbers in the case of $\mathbb{Z}$ and irreducible varieties in the case of $\mathbb{C}[t_1, \dots, t_n]$, but we will see many more commonalities.

Indeed, at the end of the course, we will see the remarkable fact that rings in which ideals factorise uniquely into prime ideals (a key property of rings of algebraic integers) correspond to smooth affine curves (central objects in algebraic geometry) under this abstract perspective.

We will largely be following the classic text on the subject by Atiyah and Macdonald, and will cover a selection of topics, including:

- Localisation;
- Tensor products;
- Noetherian and Artinian rings;
- Primary decomposition;
- Integral extensions;
- Direct and inverse limits;
- Dimension theory;
- Dedekind domains and discrete valuation rings.

Prerequisites

The only prerequisite is a first course in ring theory, such as the IB course Groups, Rings, and Modules. The Part II courses Algebraic Geometry and Number Fields provide useful motivation for the course, but are not an essential prerequisite for it. Part II Representation Theory and Galois Theory complement the course well, but are not essential either.

Literature

1. M. F. Atiyah and I. G. Macdonald. *Introduction to commutative algebra*. Addison-Wesley Publishing Co., 1969
2. A. B. Altman and S. L. Kleiman. *A term of commutative algebra*. Worldwide Center of Mathematics, LLC, 2013

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term.