Representation theory of the symmetric group (MT 2026)

Non-Examinable (Graduate Level)

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The representation theory of symmetric groups is a classical subject going back to Frobenius, with many connections across mathematics, computer science and physics. The rich interplay between algebra and combinatorics in the representation theory of symmetric groups has led to the development of a deep yet versatile theory, which has kept the topic at the forefront of modern research. Recently it has served as a blueprint for the study of so-called diagram algebras, such as the Temperley-Lieb algebra, the partition algebra, the Brauer algebra and in even more general diagrammatic settings such as the spider category.

Closely related is the representation theory of the general linear group, going back to Frobenius' greatest student, Schur. Schur linked the two theories together in a process now known as Schur-Weyl duality, which itself serves as a blueprint for similar phenomena within diagrammatic algebra. The modern treatment using Young tableaux is due to Alfred Young, Gordon James and Sandy Green.

While we work over $\mathbf{C}$, there are comparable theories in characteristic p. There are analogous (and harder) ordinary and modular theories for the type A Hecke algebra, the quantum Schur algebra and quantum GL_n.

I'll aim to cover some of the following:

- The symmetric group and its elements, combinatorics
- Irreducible modules and their parametrisation
- Counting standard tableaux I: the Young–Frobenius formula
- Counting standard tableaux II: the hook formula
- Polynomial representations of $\mathrm{GL}_n(\mathbf{C})$
- Schur-Weyl duality

Prerequisites

Basic group theory; ordinary representations/character theory from the Part II (or equivalent) course; a basic familiarity with categories helps. The Part III course on the representations of finite-dimensional complex semisimple Lie algebras is complementary to this course.

Preliminary Reading

1. C.W. Curtis, Pioneers of representation theory: Frobenius, Burnside, Schur and Brauer (AMS 1999)
2. P. Etingof *et al*, Introduction to representation theory (AMS 2011)
3. I.M. Isaacs, Character theory of finite groups (Dover reprint 1994)
4. G.D. James & M.W. Liebeck, Representations and characters of groups (CUP, 2nd edn 2001)
5. W. Ledermann, Introduction to group characters (CUP, 2nd edn, 1987)

Literature

1. C. Bowman, Diagrammatic Algebra, Springer 2025.
2. T. Ceccherini-Silberstein *et al*, Representation theory of the symmetric groups: the Okounkov–Vershik approach, character formulas and partition algebras, CUP 2010
3. W. Fulton, Young tableaux, LMS Student Texts 35, CUP 1997.
4. R. Goodman & N.R. Wallach, Symmetry, representations and invariants, GTM 255, Springer 2009.
5. J.A. Green, Polynomial representations of GL_n (with an appendix on Schensted correspondence and Littelmann paths), LNM 830, Springer 2008.
6. G.D. James, The Representation Theory of the Symmetric Groups. Lecture Notes in Mathematics 682, Springer, Berlin, 1978.
7. G. D. James & A. Kerber, The Representation Theory of the Symmetric Group. Encyclopedia of Mathematics and its applications vol. 16, Addison-Wesley, 1981.
8. A. Kleshchev, Linear and projective representations of the symmetric groups, Cambridge Tracts 163, CUP 2005.
9. The collected papers of Alfred Young, G. de B. Robinson (ed), University of Toronto Press, 1977