

Noncommutative Noetherian Rings (L24)

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This course can be viewed as a first course on noncommutative rings and their modules. From the point of view of this course the term noncommutative ring will mean not necessarily commutative associative unital ring. Perhaps the simplest examples that are not commutative are rings $M_n(F)$ of $n \times n$ matrices over fields. These examples already begin to illustrate why noncommutative rings are pervasive in mathematics though they do not illustrate well the complexity of the general theory. The condition that a noncommutative ring be noetherian can (and will) be viewed through a number of lenses. Perhaps the most illuminating in terms of the importance of the concept is that it guarantees that finitely generated modules for the ring are always finitely presented. Many noncommutative rings that arise in nature do satisfy this condition including enveloping algebras of finite dimensional Lie algebras over a field, rings of polynomial differential operators and some group algebras. In this course we will begin to develop the general theory of noetherian rings but also aim to keep examples in mind throughout. Topics that we expect to cover include

- Noncommutative form of Hilbert's Basis Theorem
- Primitive and Prime Ideals
- Artinian and Semisimple Rings
- Noncommutative localisation and Goldie's Theorem
- Filtrations, Associated Graded Rings and Completions
- Examples: Weyl Algebras, Enveloping Algebras.

Prerequisites

It will be assumed that the material in the Commutative Algebra course is familiar and an awareness of the material in Lie Algebras and their Representations may be an advantage.

Literature

1. K. R. Goodearl and R.B. Warfield, Jr. *An Introduction to Noncommutative Noetherian Rings* Second Edition, London Mathematical Society Student Texts **61** 2004.
2. J.C. McConnell, J.C. Robson, *Noncommutative Noetherian Rings* Graduate Studies in Mathematics Volume 30, American Mathematical Society, 2001.
3. L. Huishi, F. Van Oystaeyen, *Zariskian Filtrations*, Kluwer Academic Publishers 2013.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.