Lie Algebras and their representations (M24)

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This course is an introduction to the basic properties of finite dimensional complex Lie algebras and of their representations.

Lie algebras are 'infinitesimal symmetries'; linearisations of groups. They are ubiquitous in many branches of mathematics: in topology, in arithmetic and algebraic geometry, and in theoretical physics (string theory, exactly solvable models in statistical mechanics),...

One reason for their importance is that the finite dimensional complex representations of the simple Lie algebras are exactly the same as those of the corresponding groups. So instead of needing to study the topology and geometry of the simple Lie groups, or the algebraic geometry of the simple algebraic groups, we can use nothing other than linear algebra and still completely describe these representations.

(Later you will want to study the topology and geometry as well, of course!)

We will cover the following topics:

Definitions, motivations, and basic structure theory.

Root systems, Weyl groups, the finite simple Lie algebras.

Classification of finite dimensional representations, Verma modules, Weyl character formula.

Crystals, Littelmann paths.

If there is time, we will finish by discussing affine Lie algebras, the basic representation, Boson-Fermion correspondence, and theta functions.

Desirable Previous Knowledge

None other than linear algebra, but the part II course on representation theory (or equivalent) will be useful as background.

Reading to complement course material

1. V. Kac, Infinite dimensional Lie algebras, Cambridge University Press
2. M. Kashiwara. On crystal bases. *in* Representations of groups (Banff, AB, 1994), 155–197, CMS Conf. Proc., 16,
3. N. Jacobson. Lie algebras

Additional support

Exercises will be provided in class, and four examples classes will be given. There will be a one-hour revision class in the Easter Term.