Hochschild cohomology and deformation theory (L16)

Non-Examinable (Graduate Level)

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Algebraic deformation theory is primarily concerned with the interplay between homological algebra and the perturbations of algebraic structures. For example one might want to deform a commutative algebra to give a non-commutative one via 'quantisation'.

In a series of papers Gerstenhaber developed deformation theory for associative algebras. The cohomology theory required in this context is Hochschild cohomology and one finds that 2-cocycles arise from deformations. A more advanced approach is to consider additional algebraic structures, including that of a differential graded Lie algebra defined on the Hochschild cocomplex of the associative algebra. Deformations correspond to Maurer-Cartan elements.

The course will start with a brief introduction to Hochschild cohomology which is the cohomology theory for associative algebras. Possible examples to consider are the Hochschild cohomology rings of polynomial algebras and of finite group algebras. I will then discuss Gerstenhaber's work. There have been applications in a variety of contexts and I'll aim to choose one or two according to the interests of the audience. Alternatively I may finish by discussing Hopf algebras and their deformations, and consider some examples of quantum groups.

Prerequisites

I shall assume basic algebraic knowledge with experience of groups, rings, modules and commutative algebra and it will be helpful to have met Lie algebras. I will point out connections with the courses on algebraic D-modules and on modular representations of finite groups.

Preliminary Reading

The introductory article by Fox is a good place to start. It first describes the classical theory of deformations of associative algebras and then moves on to more general algebraic structures. The papers of Gerstenhaber are also very readable.

Literature

1. T. Fox *An introduction to algebraic deformation theory.* J Pure and Applied Algebra, **84**, 17-41 (1999).

2. M.Gerstenhaber *On the deformation theory of rings and algebras.* Ann. Math. **78**, 267-288 (1963).

3. M.Gerstenhaber *On the deformation theory of rings and algebras II.* Ann. Math. **84**, 1-19 (1966).

4. M.Gerstenhaber *On the deformation theory of rings and algebras III.* Ann. Math. **88**, 1-34 (1968).

5. M.Gerstenhaber *On the deformation theory of rings and algebras IV.* Ann. Math. **99**, 257-256 (1974).

6. S.F. Siegel and S.J. Witherspoon *The Hochschild cohomology ring of a group algebra.* Proc. London Math. Soc. **79**, 131–157 (1999).

7. S.J. Witherspoon, *Hochschild cohomology for algebras.* Graduate studies in mathematics **204**, A.M.S. 2019.