Group Cohomology (L16)

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Group cohomology uses methods motivated by algebraic topology to study groups and their representations. Applications appear throughout mathematics, in representation theory, topology, geometry and number theory (Galois cohomology).

I hope to cover all/most of the following:

- Definitions and resolutions.
- Low degree cohomology, derivations and group extensions.
- Group homology, the Schur multiplier (or multiplicator) and central extensions.
- Crossed products, division algebras and Brauer groups of fields.
- Cup products and cohomology rings.
- Lyndon-Hochschild-Serre spectral sequence, inflation-restriction exact sequence.
- Cohomological dimension, finiteness conditions F_n and FP_n.

The course will be similar, but not identical, to the one given last year, with a little more emphasis on spectral sequences.

Prerequisites

My approach will be purely algebraic but it will be useful to have some acquaintance with (co-)homology from other contexts. Many of the examples will be of groups arising in geometry and topology.

You should know your way about groups, rings and modules. For this, the III course on Commutative Algebra will provide good experience. It would also be useful to have met the group algebra, the associative algebra underlying group representation theory.

I shall assume acquaintance with tensor products (of vector spaces/modules and algebras), and, for the section on Brauer groups of fields, I shall also assume that you have met Galois groups (though you won't in fact need much more than the definition).

Literature

1. D.J. Benson, *Representations and Cohomdlogy, volume II, Cohomology of Groups and Modules.* Cambridge University Press, 1991.
2. K.S. Brown, *Cohomology of Groups.* Springer, 1982.
3. L. Evens. *The Cohomology of Groups.* Oxford University Press, 1991.
4. R.S. Pierce, *Associative Algebras.* Springer, 1982.
5. D.J.S. Robinson, *A Course in the Theory of Groups.* Springer, 1982.
6. J.J. Rotman, *An Introduction to Homological Algebra.* Springer, 2009.
7. C.A. Weibel, *An Introduction to Homological Algebra.* Cambridge University Press, 1994.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.