Group Cohomology (L16)

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The aim of this course is to give an introduction to group (co)homology with a focus on infinite groups.

A central observation due to Hurewicz is that for any *aspherical* topological space X, the homotopy type of X is determined by the fundamental group $\pi_1(X)$. Since any group G is the fundamental group of an aspherical space X (mainly, its $K(G, 1)$-complex), it makes sense to define the (co)homology of G as the (co)homology of the space X.

On the other hand, the (co)homology of a group G can be defined purely algebraically via projective resolutions of G-modules. This approach provides a natural framework for incorporating a wider class of coefficient modules, thereby yielding a richer theory.

In this course we will combine both the algebraic and topological viewpoints. Our aim is to develop a toolkit for computing (co)homological invariants of groups, with examples from low-dimensional topology and geometric group theory.

The course will cover some of the following topics.

- Homological algebra background.
- Group (co)homology with coefficients.
- Fox calculus.
- Lyndon-Hochschild-Serre spectral sequence.
- Cohomological dimension and finiteness properties.
- Group splittings and Mayer–Vietoris sequence.

Prerequisites

Familiarity with groups, rings and modules, as well as tensor products over non-commutative rings will be assumed. Knowledge of the content from Part II Algebraic Topology is strongly recommended.

Literature

1. K. S. Brown, *Cohomology of groups*, Graduate Texts in Mathematics, 87, Springer, New York-Berlin, 1982
2. R. Geoghegan, *Topological methods in group theory*, Graduate Texts in Mathematics, 243, Springer, New York, 2008
3. C. A. Weibel, *An introduction to homological algebra*, Cambridge Studies in Advanced Mathematics, 38, Cambridge Univ. Press, Cambridge, 1994

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.