Commutative Algebra (M24)

Dr O. Becker

This course will provide an introduction to the theory of commutative rings and modules over these rings. It should be viewed as a foundational course for Algebraic Geometry and Algebraic Number Theory.

The course will cover a selection of topics including:

- Unital commutative rings, ideals, Hilbert's basis theorem
- Tensor products, flatness, localization
- Nakayama's lemma, integral and finite extensions, going up and going down theorems
- Noether's normalization theorem and Hilbert's nullstellensatz, primary decomposition
- Direct and inverse limits, a brief introduction to completions, the Artin-Rees lemma
- Dimension theory and Hilbert functions
- Discrete valuation rings and Dedekind domains

Prerequisites

You will have attended a first course in ring theory, such as the IB course Groups, Rings and Modules. Experience of more advanced material such as Part II courses Galois Theory, Representation Theory, Algebraic Geometry or Number Fields is desirable (contributing to your level of algebraic maturity) but not essential.

Literature

1. M.F. Atiyah and I.G. Macdonald, *Introduction to commutative algebra*, Addison-Wesley, 1969.
2. J.S. Milne, *A Primer of Commutative Algebra, https://www.jmilne.org/math/xnotes/ca.html*
3. *D. Eisenbud,* Commutative Algebra with a view toward Algebraic Geometry*, Spring-Verlag, 1995.*
4. *H. Matsumura,* Commutative Algebra*, second edition, The Benjamin/Cummings publishing co., 1980.*
5. *H. Matsumura,* Commutative Ring Theory*, Cambridge Studies 8, CUP, 1989.*

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.