

Topics in Combinatorics (M16)

W. T. Gowers

This course will present a number of different results in order to showcase the wide variety of techniques, problems and results to be found in combinatorics, where “combinatorics” will be interpreted somewhat broadly. There will be an emphasis on results with proofs that surprised experts with their brevity or simplicity: often this means that the proof presented in the course will not be the first proof discovered. The content of the course will overlap substantially with the following topics.

1. Sperner’s theorem and the Erdős-Ko-Rado theorem.
2. Well-separated families of sets.
3. Székely’s proof of the Szemerédi-Trotter theorem.
4. Solymosi’s sum-product theorem.
5. Greene’s short proof of the Kneser conjecture.
6. The Marcus-Tardos theorem.
7. Entropy and the number of P3s in a bipartite graph.
8. Dvir’s solution to the finite-field Kakeya problem.
9. Croot, Lev, Pach, Ellenberg, Gijswijt, and the cap-set problem.
10. Levy’s isoperimetric inequality.
11. An approximate Ramsey theorem for colourings of high-dimensional spheres.
12. Szarek’s proof of the Kashin decomposition.
13. Huang’s proof of the sensitivity conjecture.
14. Gurvits’s simpler proof of the van der Waerden conjecture on permanents of doubly stochastic matrices.

Pre-requisites

There are few prerequisites beyond some knowledge of linear algebra and a familiarity with basic definitions connected with graphs.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.