

M. PHIL. IN STATISTICAL SCIENCE

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Friday 6 June 2003 9 to 12

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PAPER 35

QUANTUM INFORMATION THEORY

*Attempt **FOUR** questions.*

*There are **five** questions in total.*

*The questions carry equal weight.*

**You may not start to read the questions  
printed on the subsequent pages until  
instructed to do so by the Invigilator.**

**1** Let  $\mathbb{S}$  be an Abelian subgroup of the  $n$ -qubit Pauli group  $\mathbb{G}_n$ . Define a quantum stabilizer code associated with  $\mathbb{S}$  and discuss its relation to the symplectic formalism. Give an example of such a code.

If  $\mathbb{E}$  is a set of Pauli operators, state and prove the condition under which a stabilizer code is  $\mathbb{E}$ -correcting.

**2** State the Singleton bound separately for classical and quantum codes. Prove the quantum Singleton bound. [*Hint*: You may assume the subadditivity of quantum entropy and the condition for error-correction on a subset of qubits.]

**3** What is the capacity of a classical channel? Define a memoryless classical channel and give the formula for the channel capacity.

Classical bits are transmitted through a binary classical channel that preserves the bit with probability  $(1-p)$  and makes it unreadable with probability  $p$  ( a classical erasure channel). Calculate the channel capacity.

**4** Define the von Neumann entropy  $S(\rho)$  of a density matrix  $\rho$ . Prove or disprove the following facts:

(i)  $S(\rho)$  is a strictly concave function of  $\rho$ :

$$S(b\rho_1 + (1-b)\rho_2) \geq bS(\rho_1) + (1-b)S(\rho_2),$$

where  $0 \leq b \leq 1$  and  $\rho_1, \rho_2$  are density matrices (acting in the same Hilbert space), with equality if and only if  $b = 0$  or  $\rho_1 = \rho_2$ .

(ii)  $S(\rho)$  decreases when you pass to a subsystem, i.e.,

$$\max\{S(\rho_1), S(\rho_2)\} \leq S(\rho)$$

for  $\rho$  acting in a tensor product Hilbert space  $\mathcal{K}_1 \otimes \mathcal{K}_2$  and  $\rho_1 = \text{tr}_{\mathcal{K}_2}\rho$ ,  $\rho_2 = \text{tr}_{\mathcal{K}_1}\rho$ .

(iii)  $S(\rho_1) + S(\rho_2) \geq S(\rho)$ , where  $\rho, \rho_1$  and  $\rho_2$  are as in (ii).

**5** Define the entanglement-assisted capacity of a quantum memoryless channel. State the formula for the entanglement-assisted capacity. Define the binary quantum erasure channel and calculate its entanglement-assisted capacity.