Homotopies

1. Let $a\colon S^n \to S^n$ be the antipodal map. Show that $a \simeq \mathrm{id}_{S^n}$ if n is odd.
2. Show that $\mathbb{R}^n - \{0\} \simeq S^{n-1}$.

Homological algebra

1. Say as much as possible about the unknown group G and map f in the following exact sequences:

$$0 \longrightarrow \mathbb{Z}/2\mathbb{Z} \longrightarrow G \longrightarrow \mathbb{Z}/2\mathbb{Z} \longrightarrow 0$$

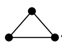
$$0 \longrightarrow G \longrightarrow \mathbb{Z} \xrightarrow{\ f\ } \mathbb{Z} \longrightarrow \mathbb{Z}/2\mathbb{Z} \longrightarrow 0$$

2. Compute the homology of the following chain complex:

$$0 \longrightarrow \mathbb{Z} \xrightarrow{\ f\ } \mathbb{Z}^2 \xrightarrow{\ g\ } \mathbb{Z} \longrightarrow 0$$

where the maps f and g are given in matrix form by $\left(\begin{smallmatrix} 2 \\ 2 \end{smallmatrix}\right)$ and $\left(\begin{smallmatrix} 1 & -1 \\ 0 & 0 \end{smallmatrix}\right)$ respectively.

Simplicial homology

1. Compute $H_n(\{\text{point}\})$ for all n.
2. Compute $H_n(S^1)$, using $S^1 \cong$ .
3. Show that $H_0(X) \cong \mathbb{Z}^{\#\text{ connected components of X}}$

Mayer-Vietoris and applications

1. Using your previous computation of $H_k(S^1)$, compute $H_k(S^n)$ for $n > 1$. *(Hint: $S^n - \{\text{point}\}$ is homotopic to a point.)*
2. Show that there is no continuous map $f\colon B^n \to S^{n-1}$ whose restriction to $S^{n-1} = \partial B^n$ is the identity.
3. Hence show that any continuous map $g\colon B^n \to B^n$ has a fixed point. *(Hint: construct a map as in the previous question by considering the ray from $g(x)$ towards x for each $x \in B^n$.)*