

Oxbridge PDE Conference 2016

MR2, Centre for Mathematical Sciences, University of Cambridge

15-16 March 2016



Convenors:

Professor Clément Mouhot, DPMMS Professor Neshan Wickramasekera, DPMMS <u>cca@maths.cam.ac.uk</u>

Conference website:

https://www.maths.cam.ac.uk/oxpde-conference-2016

All lectures will take place in MR2 Refreshments will be served in MR5

Centre for Mathematical Sciences University of Cambridge Wilberforce Road Cambridge CB3 0WA



Timetable

Tuesday 15 March

11.00am	Registration and coffee in MR5
11.25am	Opening remarks
11.30am	Professor Gregory Seregin (University of Oxford) On global weak solutions to the Cauchy problem for the Navier- Stokes equations with large \$L_3\$-initial data
12.30pm	Tobias Barker (University of Oxford) A necessary condition of possible blowup for the Navier-Stokes system in half space
1.00pm	Lunch
2.00pm	Group photo
2.15pm	Tom Begley (University of Cambridge) On Short Time Existence of Lagrangian Mean Curvature Flow
2.45pm	Professor Neshan Wickramasekera (University of Cambridge) Regularity of stable CMC hypersurfaces
3.45pm	Break
4.10pm	Dr Costante Bellettini (University of Cambridge) Compactness questions for triholomorphic maps
5.10pm	Dr Luca Spolaor (Max Planck Institute) Regularity results for semicalibrated currents
7.00pm	Dinner Strada, Trinity Street



Wednesday 16 March

9.05am	Professor Mihalis Dafermos (University of Cambridge) The linear stability of the Schwarzschild solution to gravitational perturbations
10.05am	Professor Giovanni Alberti (University of Pisa)
	Mixing properties of flows associated to divergence-free velocity fields
11.05am	Break
11.30am	Professor John Ball (University of Oxford)
	Interfaces and metastability in solid and liquid crystals
12.30pm	Jamie Taylor (University of Oxford)
	Equilibrium configurations in dense nematics
1.00pm	Lunch
2.00pm	Helge Dietert (University of Cambridge)
	Landau damping of inhomogeneous states in the Kuramoto model
2.30pm	Professor Qian Wang (University of Oxford)
	An intrinsic hyperboloid approach for Einstein-Klein-Gordon equations
3.30pm	Concluding remarks
3.35pm	Close



Abstracts

Professor Gregory Seregin (University of Oxford)

On global weak solutions to the Cauchy problem for the Navier-Stokes equations with large \$L_3\$-initial data

The aim of the talk is to discuss different definitions of solutions to the Cauchy problem for the Navier-Stokes equations with the initial data belonging to the Lebesgue space \$L_3(\mathbb R^3)\$}. This is a joint work with V. Sverak.

Tobias Barker (University of Oxford)

A necessary condition of possible blowup for the Navier-Stokes system in half space

In this talk, we will discuss the behaviour of the L_3-norm of solutions to the initial boundary value problem for the Navier-Stokes system, as time approaches possible blow up. In particular, we show that the L_3-norm must tend to infinity, for the case of the half space.

In the first part of the talk, we will discuss the differences and difficulties of the case of the half space, compared to the proof of the corresponding result in the whole space (Seregin 2012).

In the second part of the talk, we will briefly sketch the resolution to these. This is a joint work with Prof. Gregory Seregin.

Tom Begley (University of Cambridge)

On Short Time Existence of Lagrangian Mean Curvature Flow

The goal of this talk will be to give an overview of recent work, joint with Kim Moore, on a short time existence problem in Lagrangian mean curvature flow. More specifically, we consider a compact initial Lagrangian submanifold with a finite number of singularities, each asymptotic to a pair of transversely intersecting planes. We are then able to construct a smooth Lagrangian mean curvature flow, existing for positive times, that attains the singular Lagrangian as its initial condition in a suitable weak sense.



Professor Neshan Wickramasekera (University of Cambridge)

Regularity of stable CMC hypersurfaces

I will describe joint work with Costante Bellettini (Cambridge) in which we develop a regularity and compactness theory for a class of n-dimensional hypersurfaces (codimension 1 integral n-varifolds) of a smooth Riemannian manifold whose generalised mean curvature is locally in L^{p} for some p > n. Subject to appropriate variational hypotheses on their regular parts and two necessary structural conditions, we show that these hypersurfaces are smooth and have constant mean curvature (CMC) away from a closed singular set of codimension 7. Our work is motivated in part by the question of existence of a CMC hypersurface with a prescribed value of the mean curvature and the recent new proof (resulting from the combined work of Hutchinson, Tonegawa, Guaraco and the speaker) of the celebrated Amgren-Pitts--Schoen--Simon existence theorem for minimal (i.e. zero mean curvature) hypersurfaces.

Dr Costante Bellettini (University of Cambridge)

Compactness questions for triholomorphic maps

A triholomorphic map u between hyperKahler manifolds solves the "quaternion del-bar" equation du=l du i + J du j + K du k. Such a map turns out, under suitable assumptions, to be stationary harmonic. We focus on compactness issues regarding the quantization of the Dirichlet energy and the structure of the blow-up set. We can relax the assumptions on the manifolds, in particular we can take the domain to be merely "almost hyper-Hermitian": this more general setting leads to the weaker notion of "almost-stationarity", without however affecting our compactness results and it leads e.g. to gaugetheoretic applications. This is a joint work with G. Tian (Princeton).

Dr Luca Spolaor (Max Planck Institute)

Regularity results for semicalibrated currents

In this talk I will introduce the notion of semicalibrated currents and give some examples. After that I will present some regularity results, which are analogous to the ones for area minimizing currents. Finally I will try to give an idea of the proof of the following fact: the singular set of a two dimensional semicalibrated current is locally finite. In particular I will explain what are the center manifold and the frequency function, and try to explain their roles in the proof.



Professor Mihalis Dafermos (University of Cambridge)

The linear stability of the Schwarzschild solution to gravitational perturbations

I will discuss a recent proof of the full linear stability of the Schwarzschild solution in general relativity to gravitational perturbations. This is joint work with Gustav Holzegel and Igor Rodnianski. See http://arxiv.org/abs/1601.06467.

Professor Giovanni Alberti (University of Pisa)

Mixing properties of flows associated to divergence-free velocity fields

The starting point of this talk is a well-known conjecture by A. Bressan, stating that (under certain assumptions) the "mixing scale" of the flow associated to a divergence-free velocity field decays at most exponentially in time. In contrast with the fact that this conjecture has already been proved in some relevant cases (see for instance the work of G. Crippa and C. De Lellis), till very recently there were just very few examples of flows which actually exhibit such an exponential decay, all given by rather irregular velocity fields. Here I will illustrate some recent examples obtained in collaboration with G. Crippa and A. Mazzucato; these examples can then be used to prove the instantaneous loss of regularity (in the space variable) for the solutions of the continuity equation associated to (divergence-free) Sobolev vector fields.

Professor John Ball (University of Oxford)

Interfaces and metastability in solid and liquid crystals

When a new phase is nucleated in a martensitic solid phase transformation, it has to fit geometrically onto the parent phase, forming interfaces between the phases accompanied by possibly complex microstructure. The talk will describe some mathematical issues involved in understanding such questions of compatibility and their influence on metastability, as illustrated by recent experimental discoveries. For liquid crystals planar (as opposed to point and line) defects are not usually considered, but there are some situations in which they seem to be relevant, such as for smectic A thin films where compatibility issues not unlike those for martensitic materials arise.



Jamie Taylor (University of Oxford)

Equilibrium configurations in dense nematics

Onsager's classical variational model for describing liquid crystalline phases has proven to be been widely successful, although it is strictly only appropriate in dilute regimes. In this talk we will present an adjustment to Onsager's model which accounts for the effects of higher concentrations. This model is less smooth than Onsager's classical model, and in particular variations cannot be taken about non-trivial minimisers. To find an analogue of the Euler-Lagrange equation of the system, we will instead reduce the problem to an equivalent, finite dimensional saddle-point problem which is sufficiently regular to obtain a vanishing derivative condition that provides solutions to the original problem. Furthermore, some numerical results concerning the phase behaviour will be presented. This is a joint work with P. Pal y-Muhoray and X. Zheng from Kent State University.

Helge Dietert (University of Cambridge)

Landau damping of inhomogeneous states in the Kuramoto model

The Kuramoto model is a successful physical model to describe the synchronisation of oscillators. In this model we show the stability of spatially inhomogeneous states through Landau damping.

Joint work with Bastien Fernandez and David Gérard-Varet.

Professor Qian Wang (University of Oxford)

An intrinsic hyperboloid approach for Einstein-Klein-Gordon equations

Abstract not available at the request of the speaker.



Accommodation

Accommodation has been booked at Churchill College (ref 37925) for non-Cambridge speakers and Oxford attendees as requested. Breakfast will be served in the Dining Hall 7.30-9.30am. Check in is from 2.00pm, check out by 10.00am at the Porters Lodge.



Map