Oxbridge PDE Conference 2021
Centre for Mathematical Sciences, University of Cambridge
19-20 April 2021

Monday 19 April 2021

9.10am	Welcome remarks (Gui-Qiang Chen and Clément Mouhot)
9.15am	**Giuseppe Mingione (University of Parma)** **Perturbations beyond Schauder** So-called Schauder estimates are a standard tool in the analysis of linear elliptic and parabolic PDEs. They had been proved by Hopf (1929 interior case), and by Scahuder and Caccioppoli (1934, global estimates). Since then, several proofs have been given (Campanato, Trudinger, Simon). The nonlinear case is a more recent achievement from the 80s (Giaquinta & Giusti, Ivert, J. Manfredi, Lieberman). All these classical results take place in the uniformly elliptic case. I will discuss progress in the nonuniformly elliptic one. From recent, joint work with Cristiana De Filippis (Torino).
10.15am	Coffee break
10.45am	**Mihalis Dafermos (University of Cambridge)** **A proof of the nonlinear stability of the Schwarzschild black hole without symmetry** I will discuss recent joint work with Gustav Holzegel, Igor Rodnianski and Martin Taylor.
11:45am	Water break
11:50am	**André Guerra (University of Oxford)** **Compensated Compactness in General Relativity (Part I)** Einstein's vacuum equations are not closed under weak convergence and hence highly oscillatory solutions may produce non-trivial matter in the limit. In 1989, Burnett conjectured that, under appropriate assumptions, this limit is characterized by the Einstein-massless Vlasov model. The purpose of this 2-part talk (with Rita Teixeira da Costa) is to give a proof of Burnett's conjecture under some gauge and symmetry assumptions, improving previous work by Huneau—Luk from 2019. In this talk I will introduce Burnett's conjecture in General Relativity. Then I will recall classical tools from compensated compactness, including microlocal defect measures, and I will use them to describe the propagation of compactness singularities in wave map systems over a fixed Lorentzian domain.

12:30pm	Lunch
1:30pm	**Melanie Rupflin (University of Oxford)** **Lojasiewicz inequalities near simple bubble trees** In the study of (almost-)critical points of an energy functional one is often confronted with the problem that the weakly-obtained limiting object does not have the same topology. For example sequences of almost-harmonic maps from a surface will in general not converge to a single harmonic map but rather to a whole bubble tree of harmonic maps, which cannot be viewed as an object defined on the original domain. One of the consequences of this phenomenon is that one of the most powerful tools in the study of (almost-)critical points and gradient flows of analytic functionals, so called Lojasiewicz-Simon inequalities, no longer apply. In this talk we discuss a method that allows us to prove such Lojasiewicz inequalities for the harmonic map energy near simple trees and explain how these inequalities allow us to prove convergence of solutions of the corresponding gradient flow despite them forming a singularity at infinity.
2:30pm	Water break
2:35pm	**Mahir Hadžić (UCL)** **On linearly oscillating galaxies** We discuss the spectrum of spectrally stable steady states of the gravitational Vlasov-Poisson system. We prove that there generically exists a gap between the zero-eigenvalue and the bottom of the essential spectrum. Using a Birman-Schwinger type principle, we then prove a criterion for the existence of eigenvalues in this "principal" gap, which in turn correspond to pure oscillatory modes. We then discuss the implications of these results on the (in)validity of linear Landau damping in this context. Our result highlights the usefulness of action-angle variables in the presence of trapped particle trajectories. Joint work with G. Rein and C. Straub.

Tuesday 20 April 2021

9.15am	**Benjamin Fehrman (University of Oxford)** **Non-equilibrium fluctuations in interacting particle systems and conservative stochastic PDE** Interacting particle systems have found diverse applications in mathematics and several related fields, including statistical physics, population dynamics, and machine learning. The large-scale behavior of these systems is essentially deterministic, and is characterized by the solution to a nonlinear diffusion equation. However, the particle process does exhibit large fluctuations away from its mean. Such deviations, though rare, can have significant consequences---such as a concentration of energy or the appearance of a vacuum---which make them important to understand and simulate. In this talk, which is based on joint work with Benjamin Gess, I will introduce a continuum model for simulating rare events in certain particle systems. The model is based on an approximating sequence of stochastic partial differential equations with nonlinear, conservative noise. The solutions capture to first-order the central limit fluctuations of the particle process, and they correctly simulate rare events in terms of a large deviations principle. We will first discuss these probabilistic concepts in some simpler settings, and later we will show that they lead to a number of interesting questions in analysis. In particular, the results rely on a detailed treatment of the associated skeleton equation---a degenerate parabolic-hyperbolic PDE with irregular drift---in energy critical spaces.
10.15am	Coffee break
10.45am	**Michele Coti-Zelati (Imperial College)** **Nonlinear inviscid damping and shear-buoyancy instability in the two-dimensional Boussinesq equations** We investigate the long-time properties of the two-dimensional inviscid Boussinesq equations near a stably stratified Couette flow. We prove that the system experiences a shear-buoyancy instability: the density variation and velocity undergo inviscid damping while the vorticity and density gradient grow. The result holds at least until a natural, nonlinear timescale. Notice that the density behaves very differently from a passive scalar, as can be seen from the inviscid damping and slower gradient growth. The proof relies on several ingredients: (A) a suitable symmetrization that makes the linear terms amenable to energy methods and takes into account the classical Miles-Howard spectral

	stability condition; (B) a variation of the Fourier time-dependent energy method introduced for the inviscid, homogeneous Couette flow problem developed on a toy model adapted to the Boussinesq equations, i.e. tracking the potential nonlinear echo chains in the symmetrized variables despite the vorticity growth.
11:45am	Water break
11:50am	**Rita Teixeira da Costa (University of Cambridge)** **Compensated Compactness in General Relativity (Part II)** Einstein's vacuum equations are not closed under weak convergence and hence highly oscillatory solutions may produce non-trivial matter in the limit. In 1989, Burnett conjectured that, under appropriate assumptions, the limit is characterized by the Einstein-massless Vlasov model. The purpose of this 2-part talk (with André Guerra) is to give a proof of Burnett's conjecture under some gauge and symmetry assumptions, improving previous work by Huneau--Luk. In this part, I explain how the gauge and symmetry assumptions allow us to reduce the study of Einstein's equations, a quasilinear system, to the study of a semilinear wave maps equation where the metric on the domain is allowed to oscillate. In the previous talk, the case of non-oscillating metrics was fully characterized. Hence, I conclude the proof by explaining how to deal with contributions from the metric oscillations.
12:30pm	Lunch
1:30pm	**Edriss Titi (University of Cambridge)** **The Inviscid Primitive Equation and the Effect of Rotation** Large scale dynamics of the oceans and the atmosphere is governed by the primitive equation (PE).It is well-known that the three-dimensional viscous PE is globally well-posed in Sobolev spaces. In this talk, I will discuss the ill-posedness in Sobolev spaces, the local well-posedness in the space of analytic functions, and the finite-time blowup of solutions to the three-dimensional inviscid PE with rotation (Coriolis force). Eventually, I will also show, in the case of “well-prepared” analytic initial data, the regularizing effect of the Coriolis force by providing a lower bound for the life-span of the solutions which grows toward infinity with the rotation rate. The latter is achieved by a delicate analysis of a simple limit resonant system whose solution approximates the corresponding solution of the 3D inviscid PE with the same initial data.
2:30pm	Water break

2:35pm	**Pierre Degond (CNRS and Imperial College)** **Moment method for rod-like polymers** There are two methods to derive hydrodynamic equations from kinetic equations when the Knudsen number tends to zero: the Hilbert method and the moment method. While the former only requires mild properties on the linearized collision operator, the latter requires specific conservation relations to hold. For classical models, the two methods are equivalent (at least formally), as the required properties of the linearized collision operator can be related to conservations. However, there are cases where conservation relations are lacking and only (so far) the Hilbert method is applicable. In this talk, I will focus on one of these examples, the kinetic model of rod-like polymers, or Doi model. I will show that suitable generalized conservation relations (aka generalized collision invariants) hold and make the moment method applicable in spite of the lack of conservation relations in the strict sense. It could lead to a better understanding of the structural properties of the Doi model and open the way to rigorous convergence proofs that would require less regularity than the Hilbert method.
3:35pm	Closing remarks (Neshan Wickramasekera)
3:40pm	Post-conference happy hour

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Organisers

Professor Clément Mouhot, University of Cambridge

Professor Neshan Wickramasekera, University of Cambridge

Professor Gui-Qiang Chen, University of Oxford

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All lectures will take place on Zoom

Zoom link to be circulated by e-mail to registered participants

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Conference website

https://www.maths.cam.ac.uk/postgrad/cca/oxbridge-pde-conference-2021