

# The effects of passive and active internal forces on the beating patterns of sperm flagella

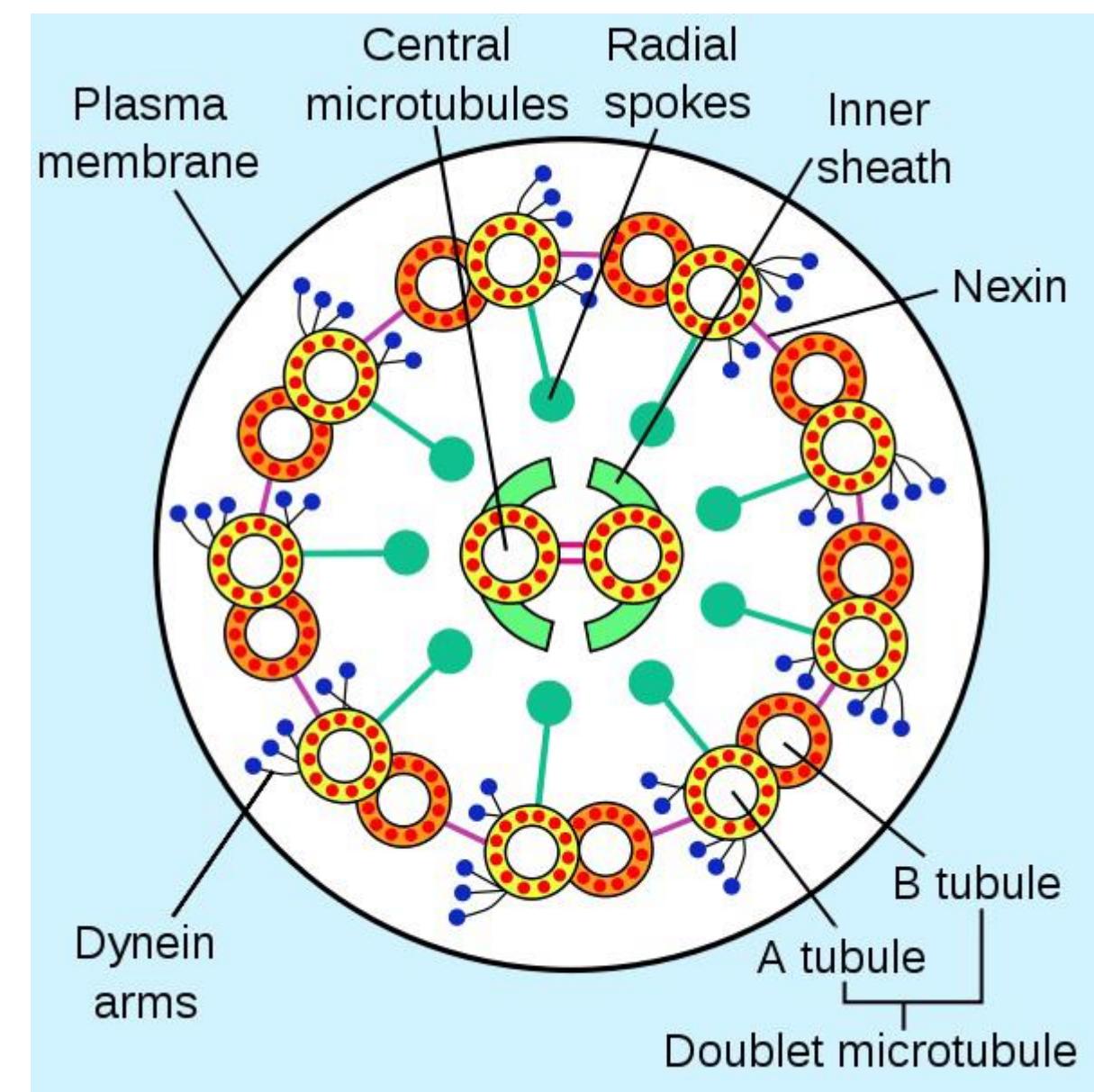
Rachel H. Coy and Tian Feng See

## Introduction: the internal structure of the axoneme

Sperm use beating flagella to transport fluid and achieve motility. We aim to extend upon existing models for the motion of such flagella by incorporating more detail related to the structure of the **axoneme**- the central cytoskeletal core of sperm flagella.

Within the cylindrical axoneme of sperm, 9 doublet microtubules are arranged around 2 inner microtubules. Microtubules are long cylindrical structures composed of polymers of tubulin. The outer doublets are connected by elastic links called nexin.

Figure : Cross section of the axoneme

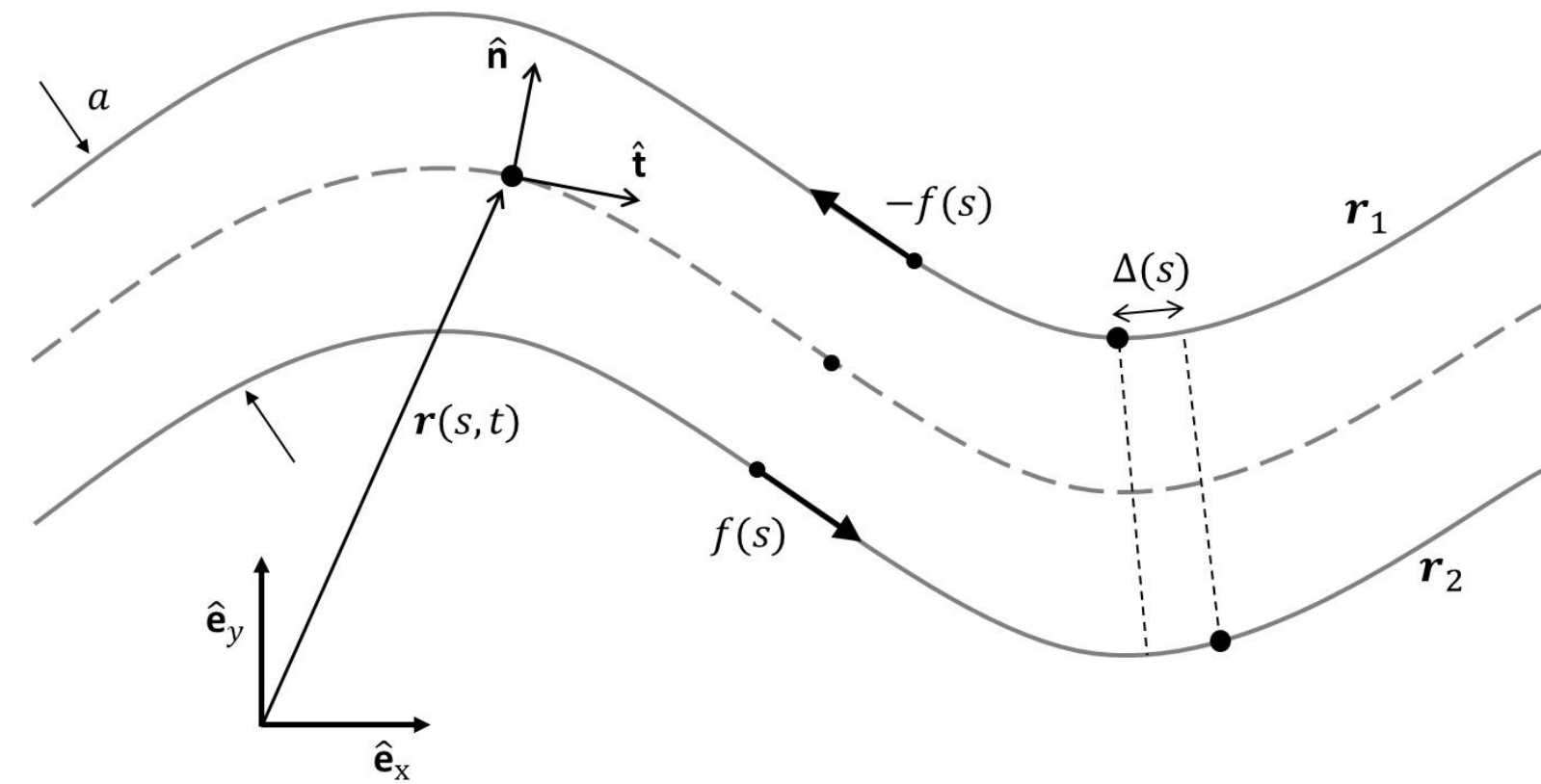


Dyneins are motor proteins that also work as the facilitator of coordinated movement and bending of the doublets and axoneme. Dynein arms extend from the A tubule of one doublet to the B tubule of an adjacent doublet, and, when attached, allow doublets to slide relative to each other. The structural restraints of the axoneme prevent doublets from sliding completely apart and result in bending of the axoneme.

## Modelling flagella as two parallel filaments

Due to the symmetry of the axoneme, it is convenient to model sperm flagella as two filaments, with constant separation width  $a$ . The shear force density produced by the internal structure of the axoneme is  $f(s, t)$ . By considering forces and moments, we can derive an equation of motion in the angle  $\alpha(s, t)$  between  $\hat{t}$  and  $\hat{e}_x$ . After solving, the beating patterns can then be plotted and analysed.

Figure :  $\mathbf{r}(s, t)$  is the position of a point which is an arclength  $s$  along the neutral central line of the flagellum (dashed curve) at a time  $t$ , relative to the fixed frame  $\{\hat{e}_x, \hat{e}_y\}$ . The internal shear force  $f(s, t)$  acts tangentially and in opposite directions on the sliding filaments  $\vec{r}_1$  and  $\vec{r}_2$ .



Sponsored by Bridgwater Summer Research Programme

## The passive filament case

Due to geometrical constraints, when the flagellum bends the two filaments travel different distances. The difference in arclength of the two filaments can be expressed as a function  $\Delta(s)$ : we assume that the internal shear force density is proportional to this quantity,  $f(s) = k\Delta(s)$ . The linearised equation of motion can be solved using boundary conditions relating to force and torque at either end of the flagellum:

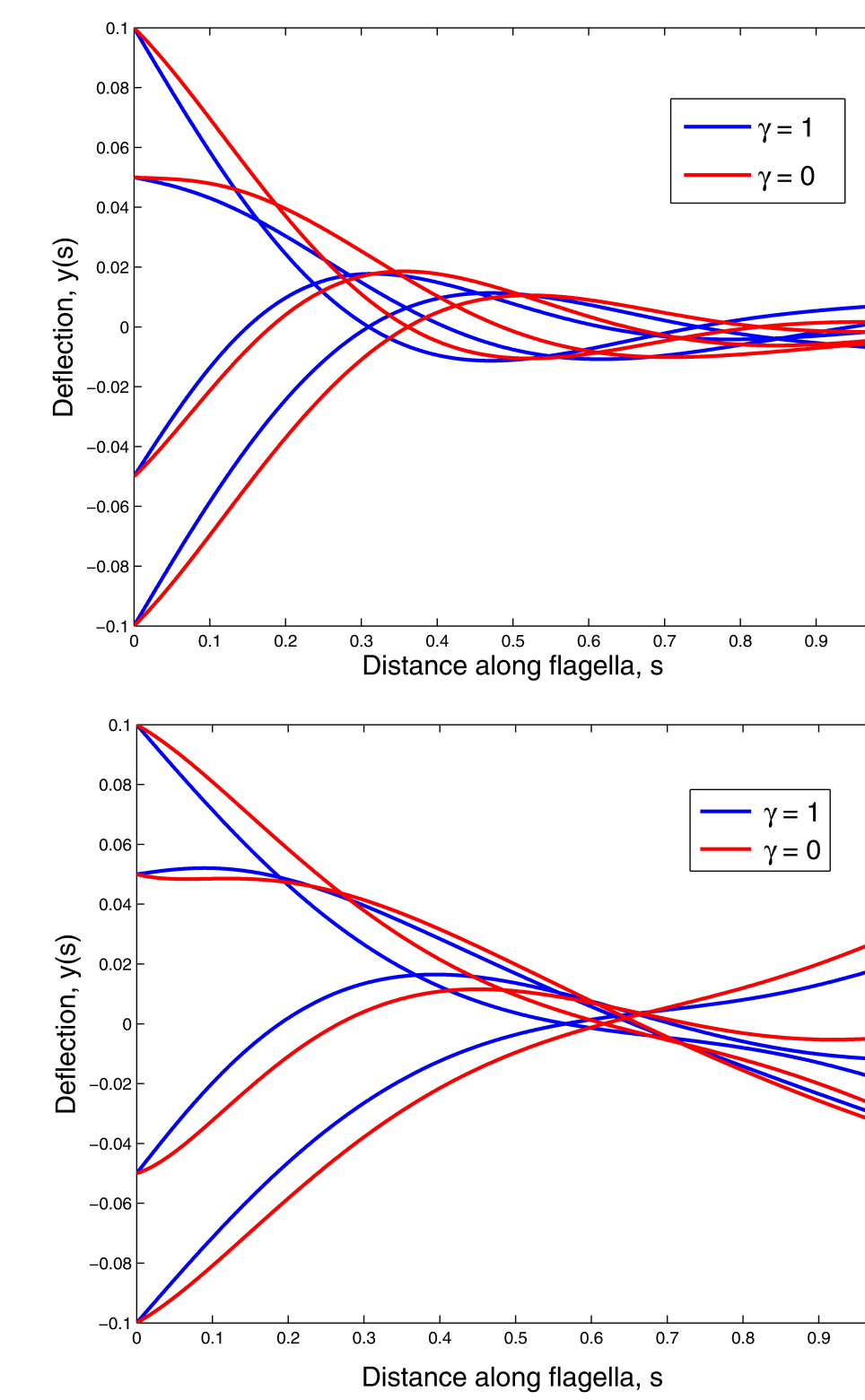
$$\ddot{\alpha} - \mu \ddot{\alpha} = iSp^4 \alpha$$

$Sp$  is the *sperm compliance parameter*, characterising the relative importance of elastic forces to viscous drag.

## The optical trap experiment

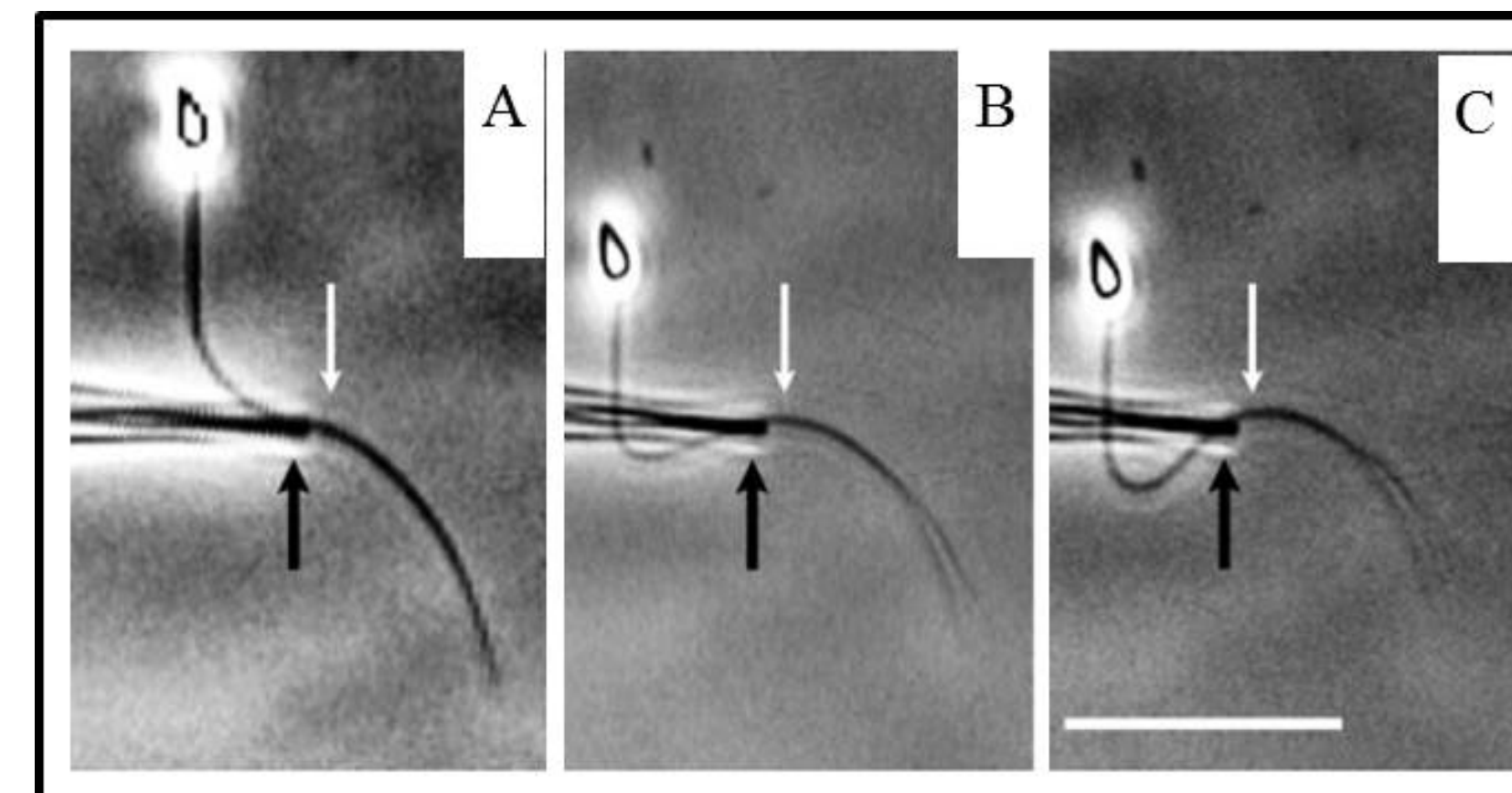
The flagellum is modelled with a given position at  $s = 0$ , the base, to mimic an experiment in which a filament is optically trapped and wiggled.

Figure : Comparison of beating patterns for flagella with different coefficients of internal forcing,  $k$ . The value of  $k$  is smaller in the first plot, corresponding to a smaller amount of force.  $\gamma = 1$  corresponds to zero basal sliding resistance;  $\gamma = 2$  corresponds to rigid anchoring at the base.



By varying parameters and considering solutions in this manner, we hope to identify a dynamical version of the *counterbend phenomenon*; this is where the induction of curvature in one part of a passive flagellum induces a compensatory countercurvature elsewhere.

Figure : The counterbend phenomenon for a static flagellum



## Geometric clutch hypothesis

The key idea of the hypothesis as proposed by Lindemann is that dyneins form bridges with a probability proportionate to the interdoublet spacing. Since the transverse force (**t-force**) is responsible for regulating the interdoublet spacing, it is the primary determinant of dynein activation and deactivation, and is consequently responsible for initiating and terminating episodes of microtubule sliding.

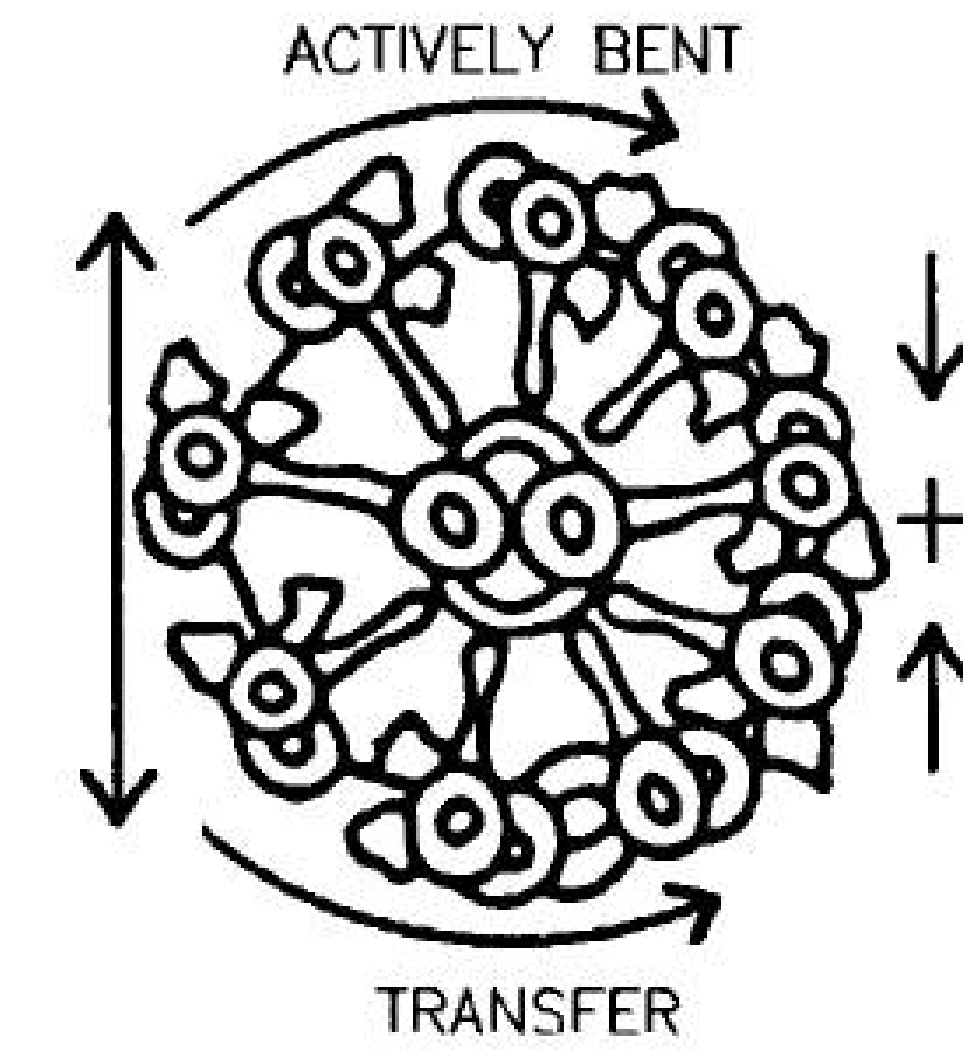


Figure : **T-force** Attachment of bridges on one side of the axoneme decreases the probability of bridging on the opposite side due to effect of transfer of forces through the interdoublet links.

In his papers, the total t-force is the sum of the local t-force and the global t-force where the local t-force is the longitudinal component of the elastic force,  $F_L$  caused by stretching of nexins and

$$\text{Global t-force} = \dot{\alpha} \times F_{total}$$

with

$$F_{total}(n) = \sum_{i=n}^{30} (F_L(i) + F_A(i))$$

the total longitudinal force on a doublet where  $F_A$  = active (longitudinal) pulling force of the dynein bridge. This total t-force is then fed into the algorithm for activation/termination of dynein bridges together with the consideration of force transfer between doublets and the effect of bridge adhesion.

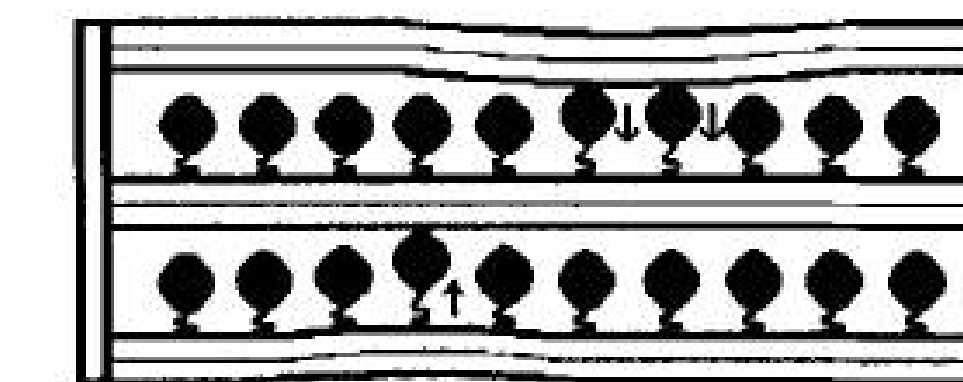


Figure : **Bridge adhesion** As bridges attach, they pull adjacent doublets slightly closer, increasing the chance that neighbouring dynein heads will also attach. It is also true conversely.

However, this formulation has a critical error. The total longitudinal force on a doublet should be

$$F_{total} = \hat{t} \cdot \int_s^L \vec{f} ds'$$

where  $\vec{f} = f_{\parallel} \hat{t} + f_{\perp} \hat{n}$ . Hence, to proceed with a more rigorous model of the hypothesis, we extend the previous model on passive filament to incorporate the ideas of the geometric clutch hypothesis. Firstly, the width  $a = a(s, t)$  changes according to Hooke's law with an unknown elastic constant in the normal direction. The shear force density will then be modified to be the sum of the passive contribution,  $f$  and also the active contribution,  $f_{active}$  where  $f_{active}$  is some probability distribution dependent on  $a$  together with the effect of force transfer and bridge adhesion. The governing equations will then be solved numerically.