UNIVERSITY OF CAMBRIDGE
Centre for Mathematical Sciences
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Modelling Cellular Kinematics in Self-Similar Plant Growth

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Motivation

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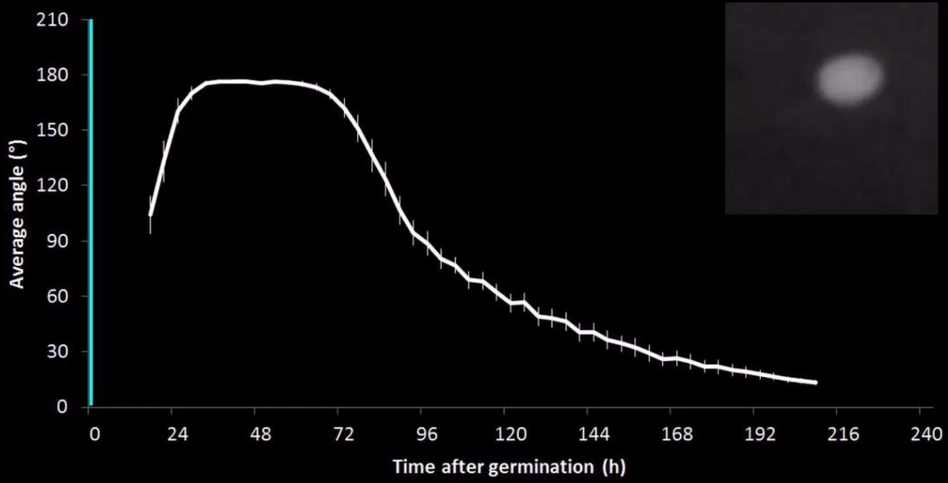
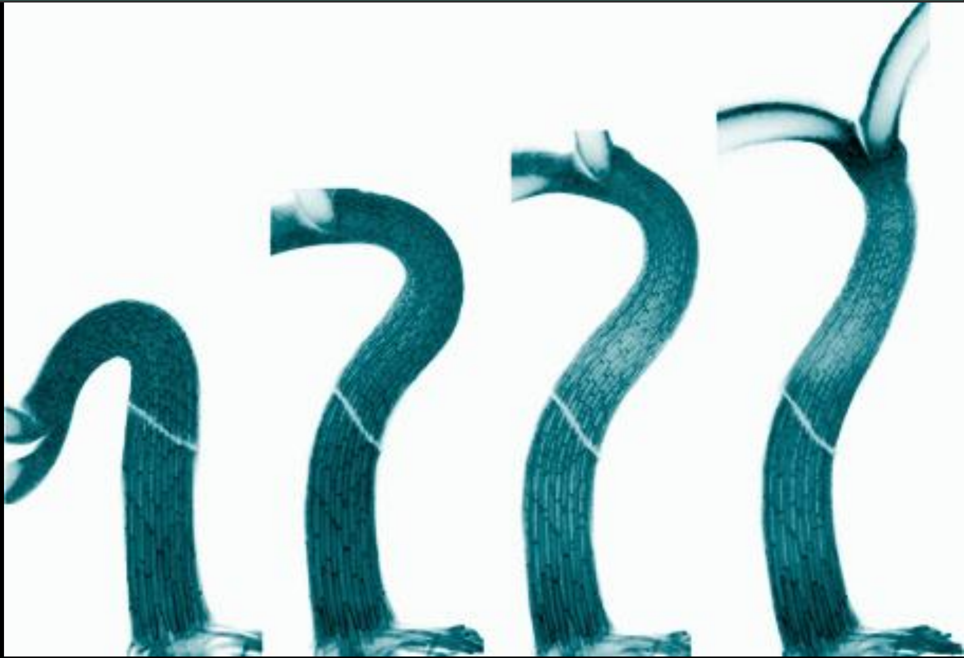
Kristoffer Jonsson

- How does a plant grow?
- What are the mechanisms behind morphogenesis?
- System is difficult to model.
- Combine curvature information with regulatory information.



Aims

- Investigated the coupling between genetic regulation and growth patterns.
- Examine structures that maintain constant geometries.
- Use concepts from differential geometry to model such structures.



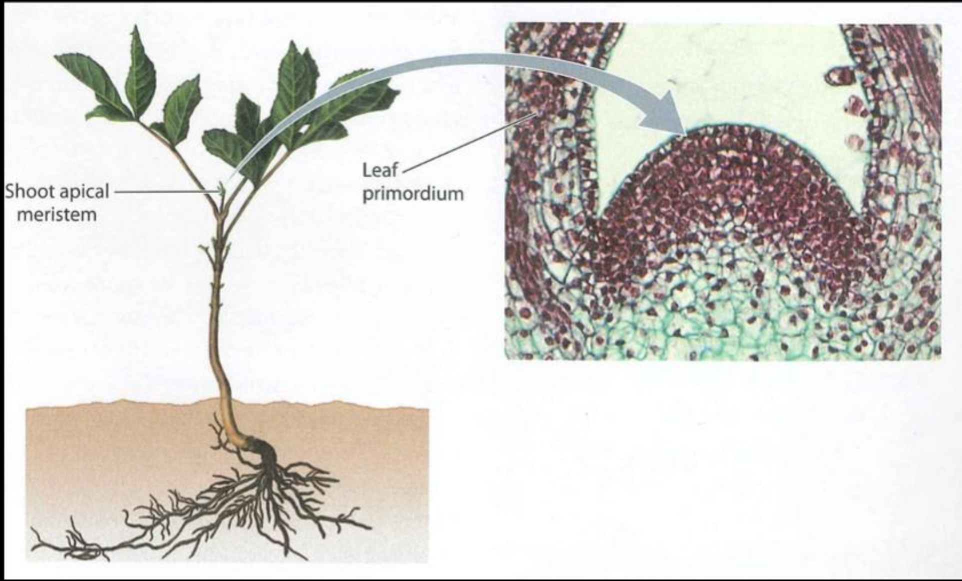
Abbas M, Alabadí D, Blázquez MA. Frontiers in plant science, 2013. Courtesy of Dr Javier Gallego-Bartolomé. (Modified)

Overview

- Biology
  - Plant Structures
  - Gene Regulatory Network
- Mathematics
  - Maps and Manifolds
  - Metric Dynamics
- Physics
- Methods
- Results
  - Models
  - Parameter Space Search

Background - Biology

- Shoot Apical Meristem (SAM).
- Undifferentiated stem cell source.
- Creates all above ground plant features.
- Almost axisymmetric.
- Can be approximated by a parabolic map.



Moore J, Jennings D. VCU BIOL 152: Introduction to Biological Sciences II. [Internet]. Richmond (VA): Virginia Commonwealth University; [cited 2025 Aug 13]. Available from: https://viva.pressbooks.pub/introbio2/

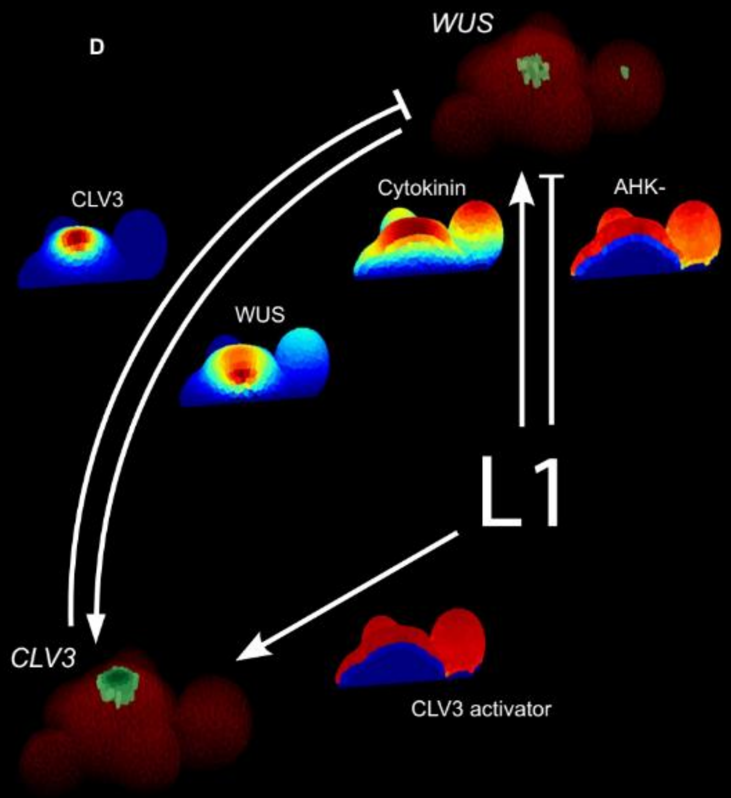
Background - Biology

- Apical Hook.
- Curved structure present in the early stages of dicot seedlings.
- Curves to push through soil and protect the SAM at the tip.
- How does it know how to curve and when to straighten?

Huang, et al. The Plant Cell, 2020.

Background - Biology

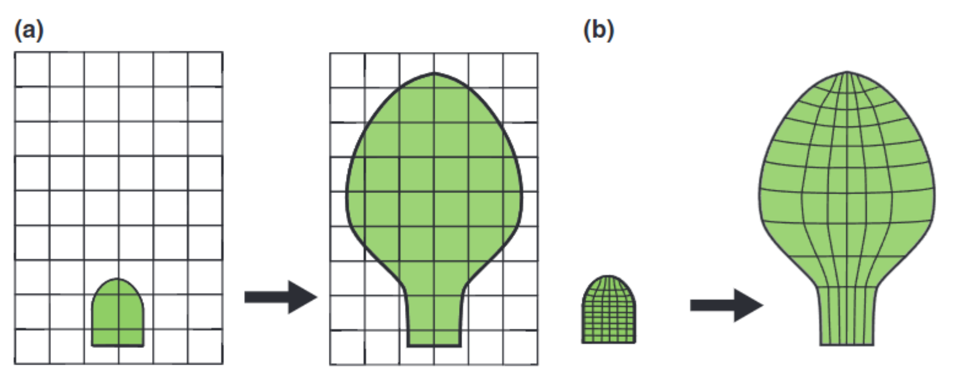
- A gene regulatory network (GRN) is a system that controls gene expression.
- Consists of activations (-->) and inhibitors (--|).
- Controls cell function based on concentration profiles.



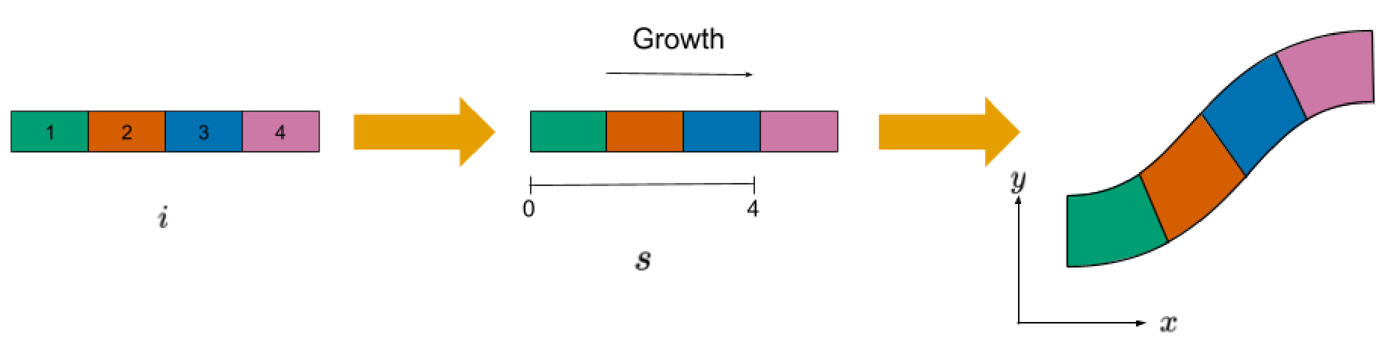
Gruel, et al. Science advances, 2016.

Background - Mathematics

- Transformed from Lagrangian to curvilinear to Eulerian coordinates.
- Lagrangian: cell indices (i,j).
- Curvilinear: constant ruler before the map (s,r).
- Eulerian: ruler after the map (x,y).



(Above) Eulerian (a) and Lagrangian (b) coordinates. Prusinkiewicz P, Runions A. New Phytologist, 2012.



(Right) The sequence of maps for a 1D line of cells.

Background - Mathematics

- Orthonormal basis so the metric in the Eulerian (lab) frame is diagonal:

$$g_{ij} = h_j^2 \delta_{ij}$$

- Metric changes by performing a Lie derivative along the *flow* of the cells with velocity *v*:

$$(\mathcal{L}_v g)_{ij} = 2\nabla_{(i} v_{j)}$$

Background - Physics

- GRNs are PDEs.
- Diffusion equation for concentration *U* with linear growth and interactions.

$$\frac{\partial U_i}{\partial t} + as \cdot \frac{\partial U_i}{\partial s} = D\nabla^2 U_i + p - gU_i + \sum_j R_{ij}(U_i, U_j)$$

Evolution: $\frac{\partial U_i}{\partial t}$

Growth: $as \cdot \frac{\partial U_i}{\partial s}$

Diffusion: $D\nabla^2 U_i$

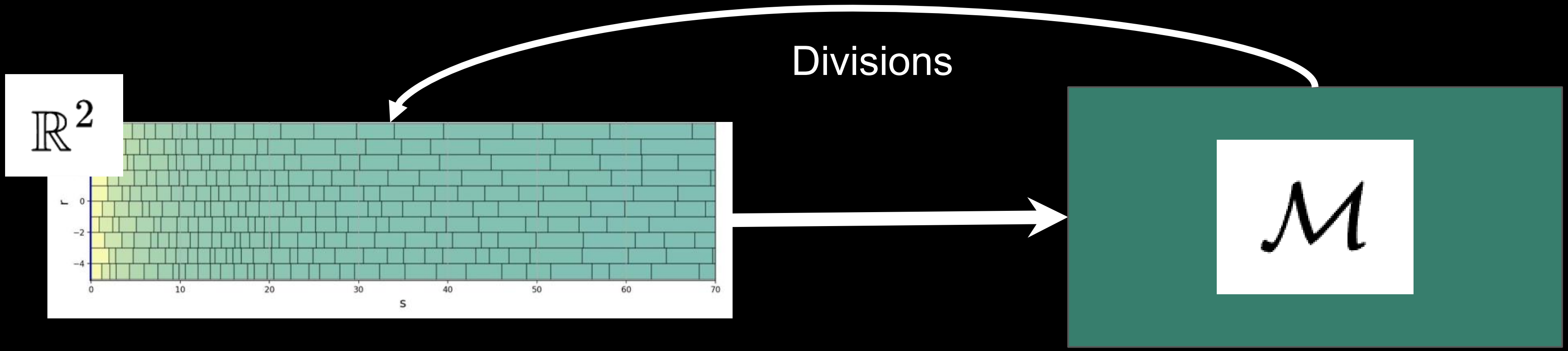
Source: p

Degradation: gU_i

Interactions: $\sum_j R_{ij}(U_i, U_j)$

Method

- Custom numerical solver for plant morphodynamics: Organism.
- Initial configuration + GRN → System Evolution
- Evolves both concentrations and topological variables.
- Map between curvilinear and Eulerian space.

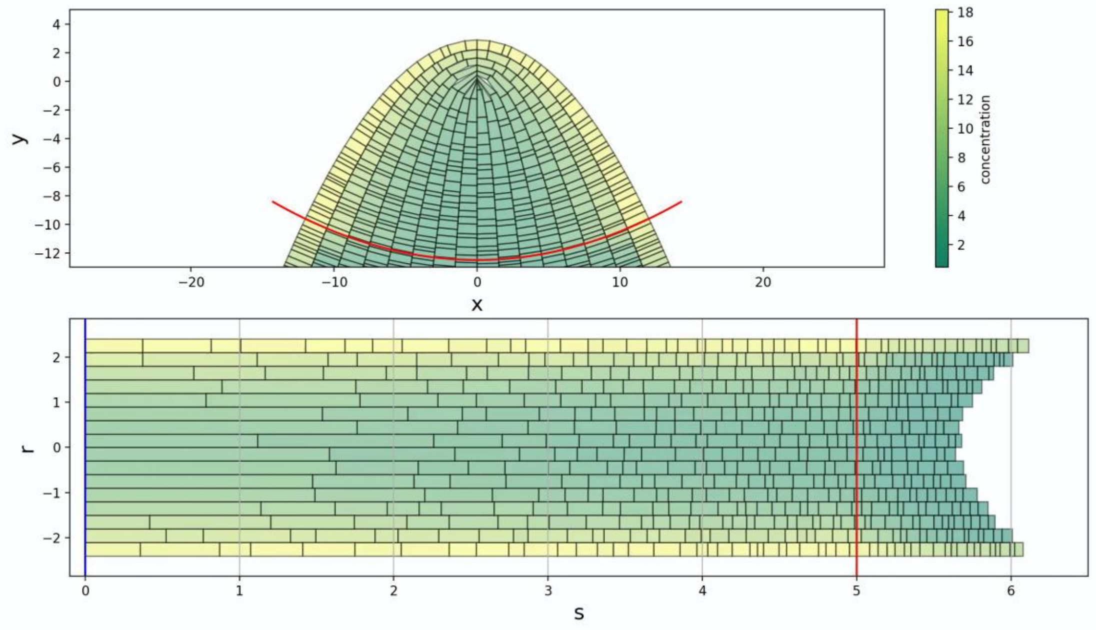


Results - SAM

- Flow of cells contained within a fixed envelope.

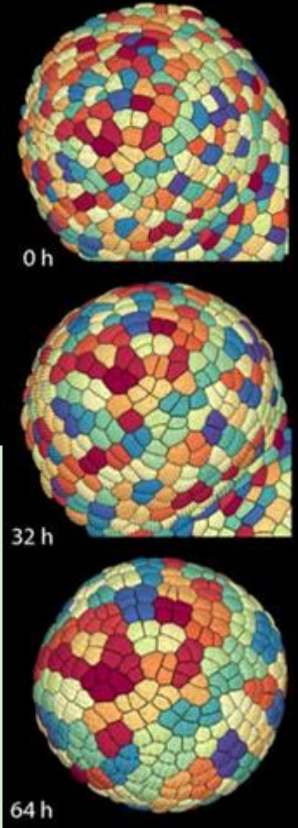
$$s \mapsto x = sr$$

$$r \mapsto y = \frac{1}{2}(r^2 - s^2)$$



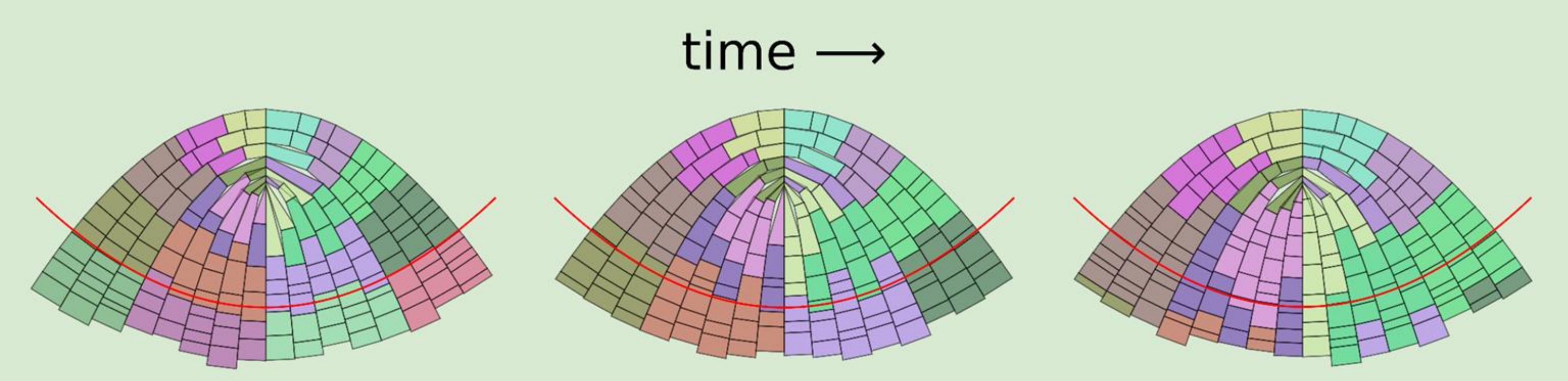
Results - SAM

- New cells from divisions coloured the same as their parent.



(Right) Confocal images, top down view. Gruel, et al. Science advances, 2016.

(Below) Simulation, side cross-section.



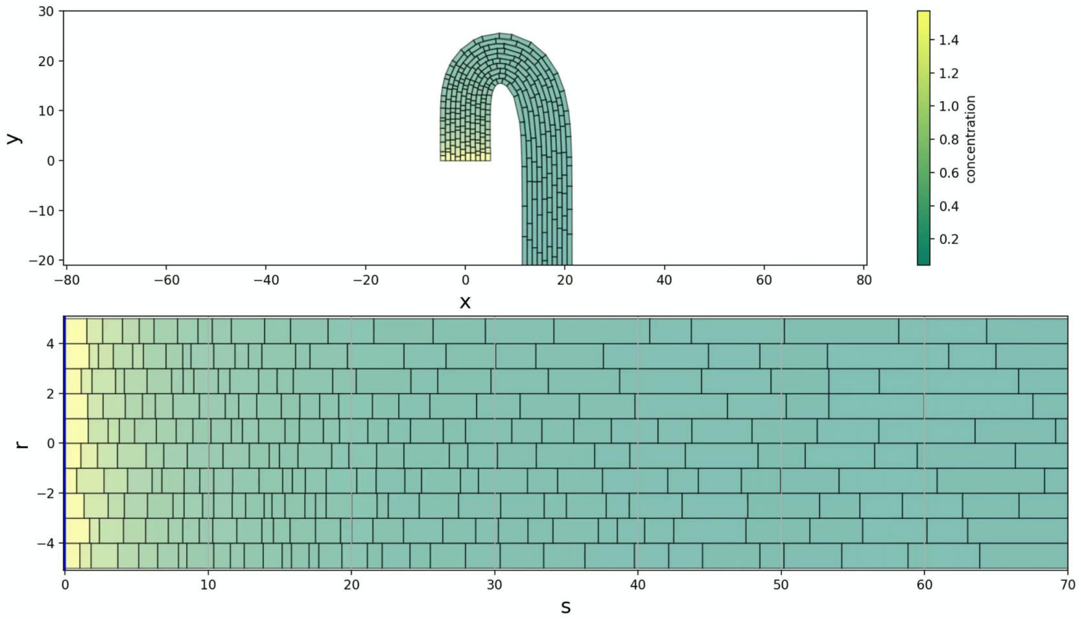
Results - Hook

- Similarly retains the overall shape.

$$\theta(s) = \int_0^s d\tilde{s}\, \kappa(\tilde{s})$$

$$x(s,r) = \int_0^s d\tilde{s}\, \sin(\theta(\tilde{s})) + r\cos(\theta(s))$$

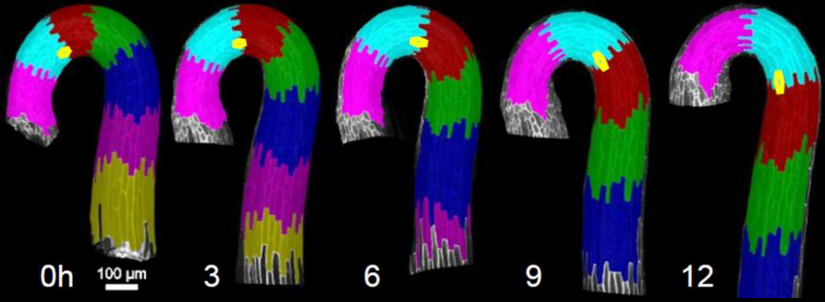
$$y(s,r) = \int_0^s d\tilde{s}\, \cos(\theta(\tilde{s})) - r\sin(\theta(s))$$

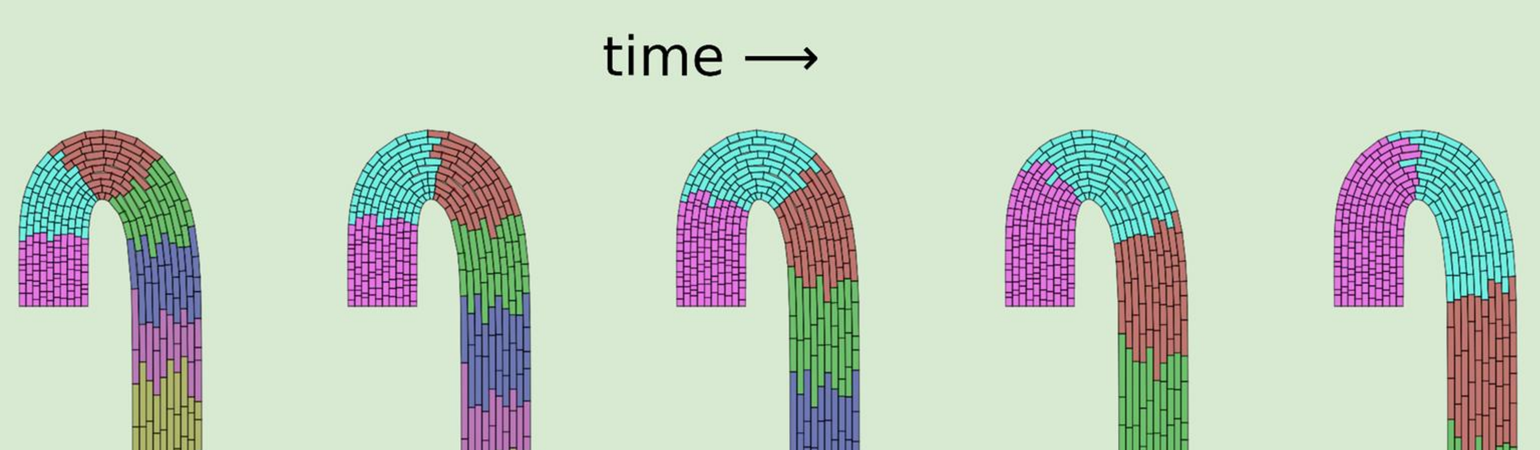


Results - Hook

- Matches confocal images.
- Flow from the tip down the stem.

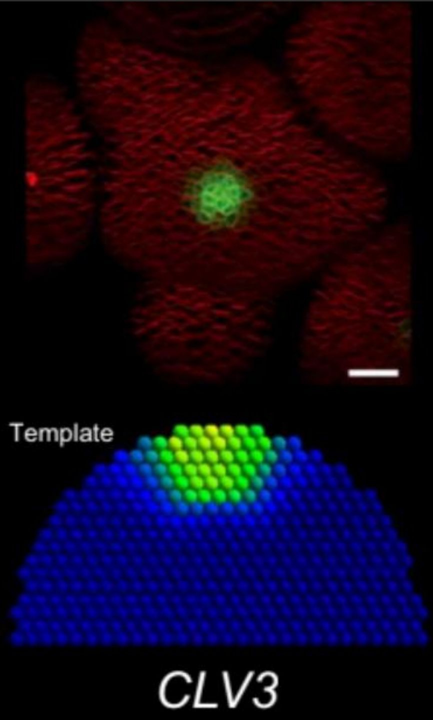
(Right) Image courtesy of an unpublished collaboration with Jonsson K, Alimchandani V, Meroz Y and Routier-Kierzkowska AL.





Results - Patterns

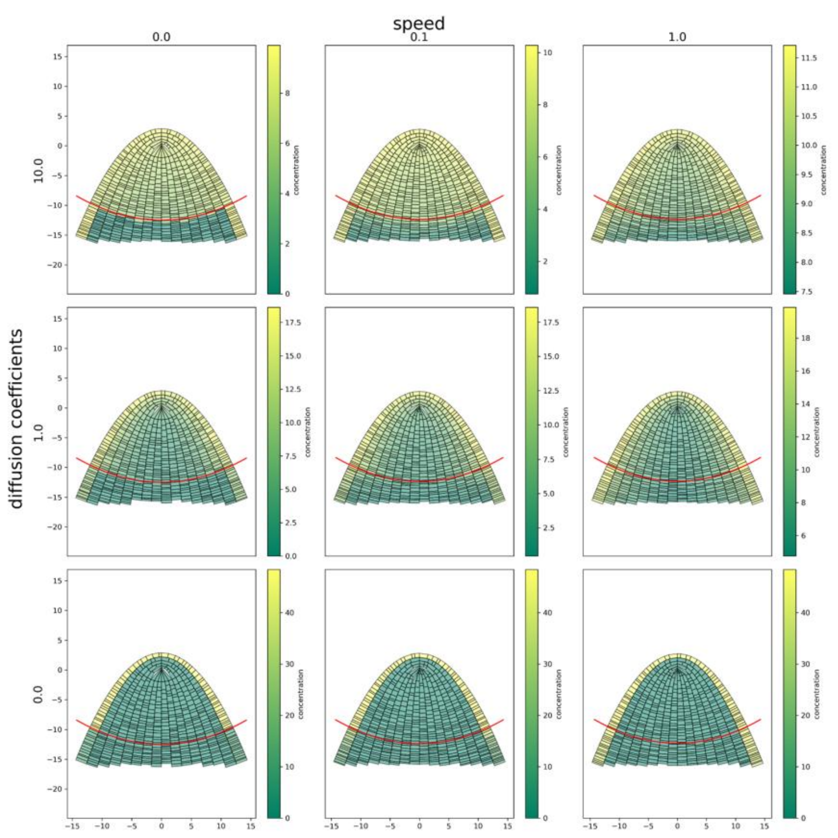
- Want to replicate patterns from experiment and in static simulations.
- Can then couple this to cell growth in the orthogonal (*r*) direction.



Gruel, et al. Science advances, 2016.

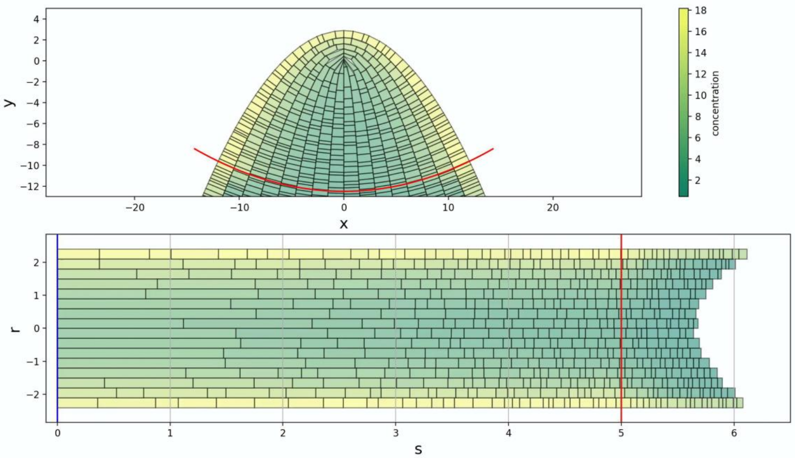
Results - Parameter Space

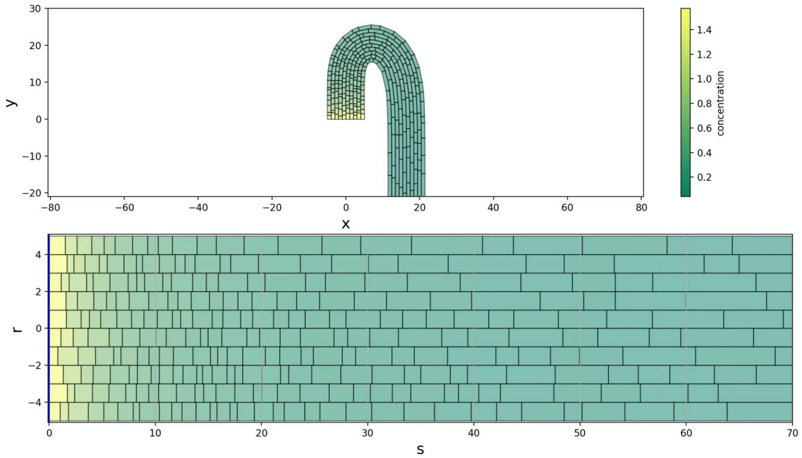
- Varying the length scale $\sqrt{\frac{D}{g}}$ and velocity coefficient *a*.
- Aim to get a spot on the top and use this to control the growth of the SAM.



Conclusion

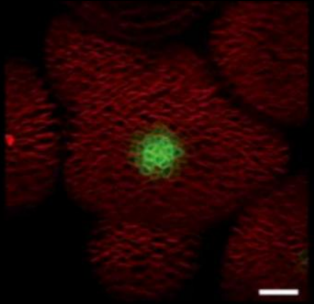
- Emulated the SAM and apical hook using the Organism code.
- Maped between a flat plane and manifold.
- Kept the shape constant.
- Investigated pattern formation using a GRN on the SAM.

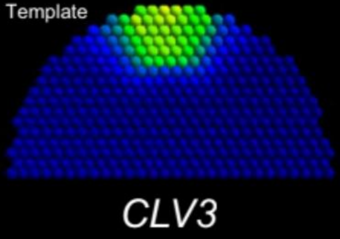




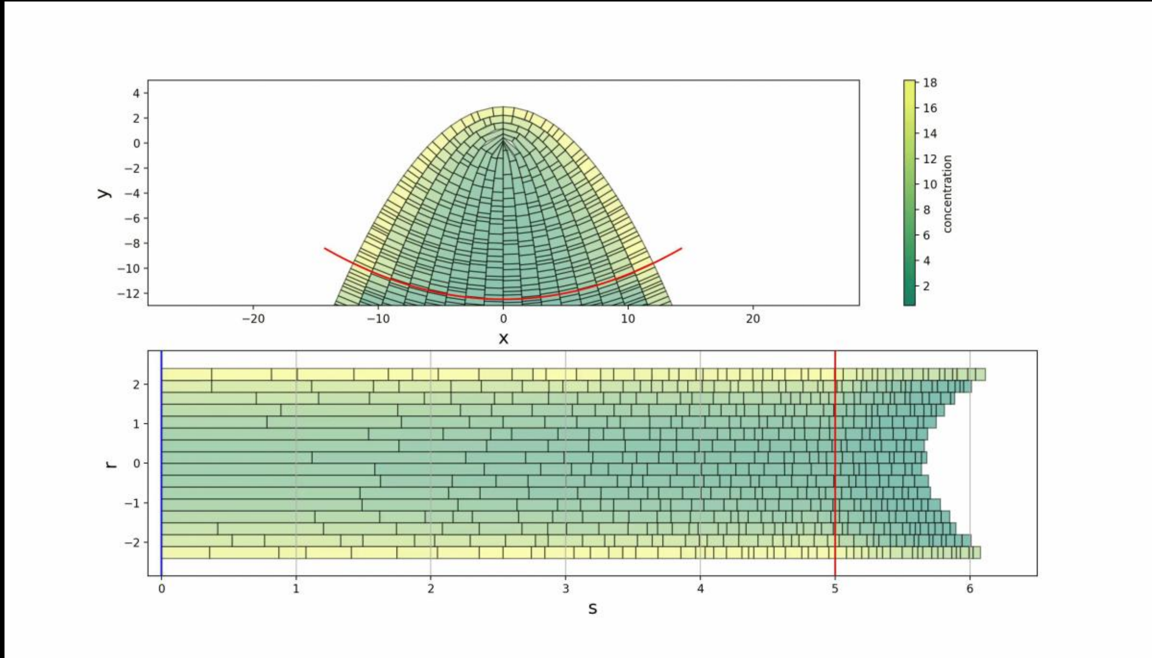
Further Work - SAM

- Spot formation on the tip of SAM.
- Couple this to growth mechanics.





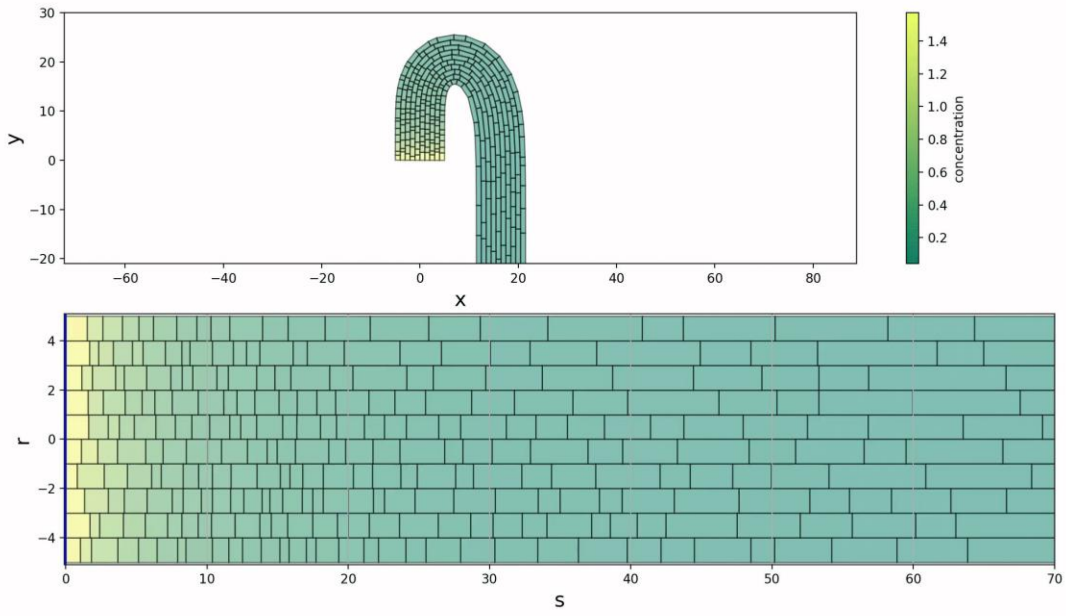
Gruel, et al. Science advances, 2016.



Future Work - Apical Hook

- Curvature update of the hook.

$$\kappa = \kappa(s,t) \Rightarrow \theta = \theta(s,t)$$



Thank you for listening.