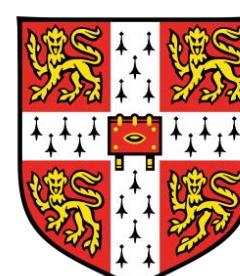


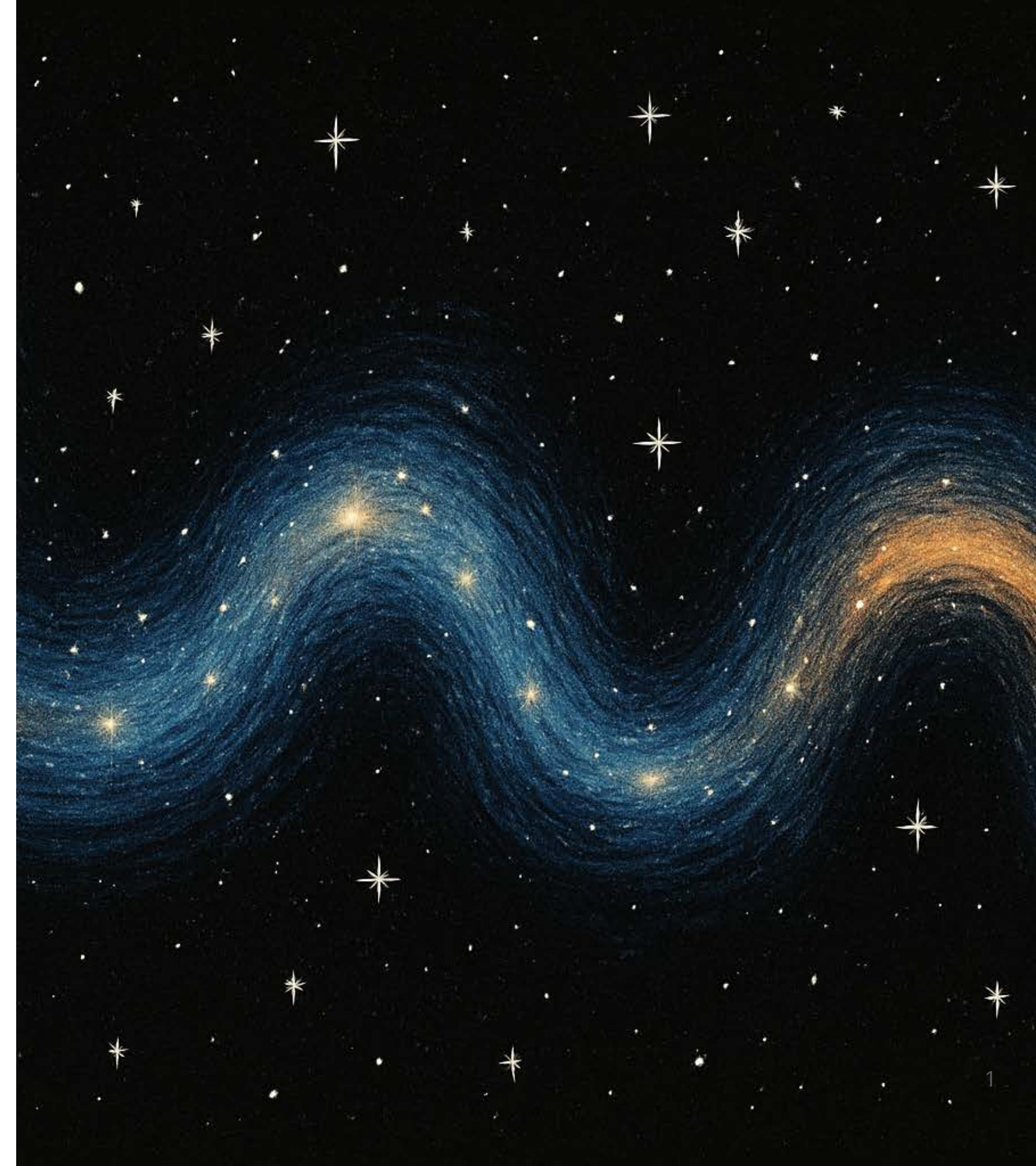
Nested Sampling for ARIMA Model Selection : A Novel Approach to Astronomical Time Series Analysis

Cambridge Mathematics Placement 2025

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Supervisor : Dr. Will Handley

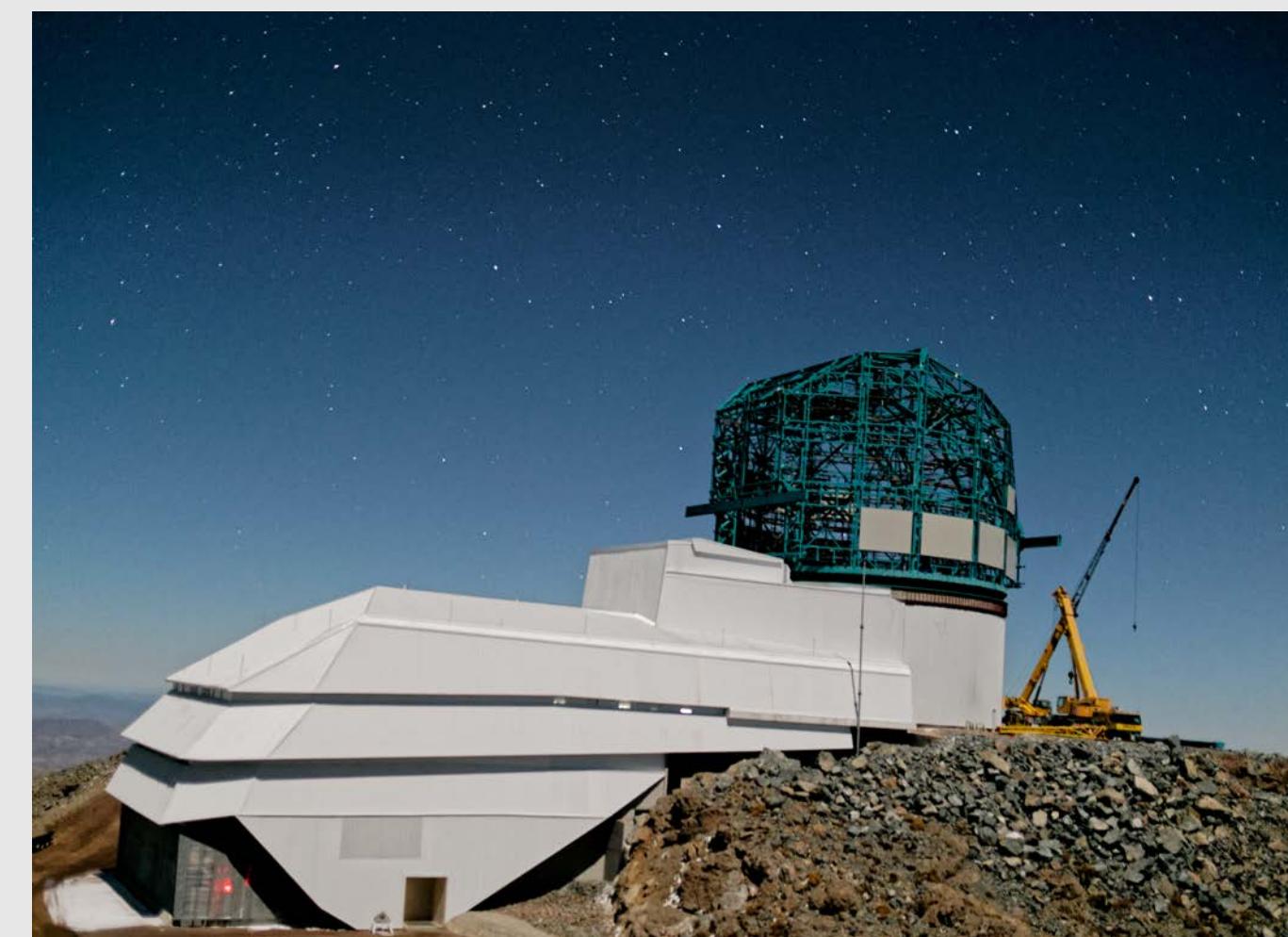


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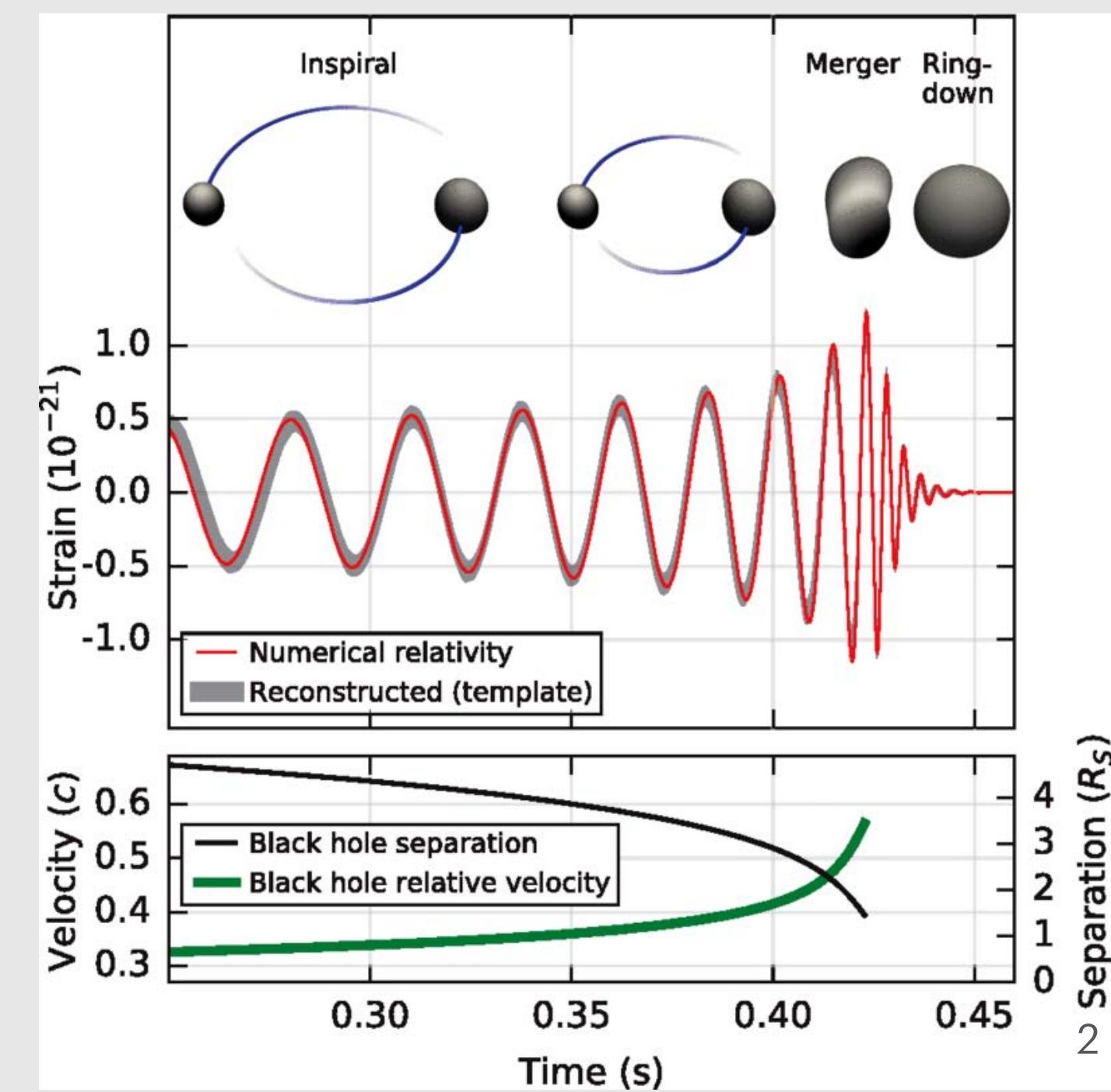
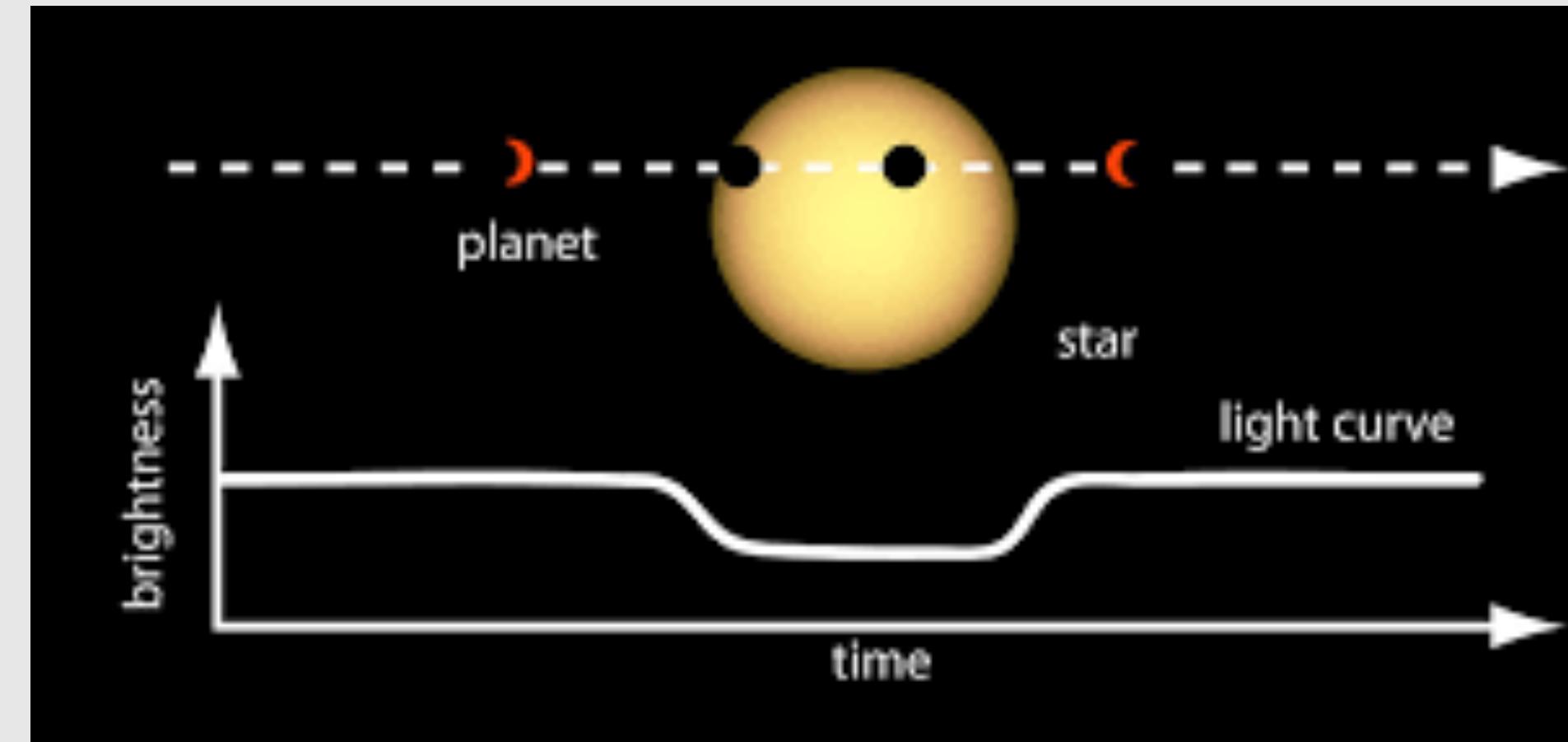


Introduction

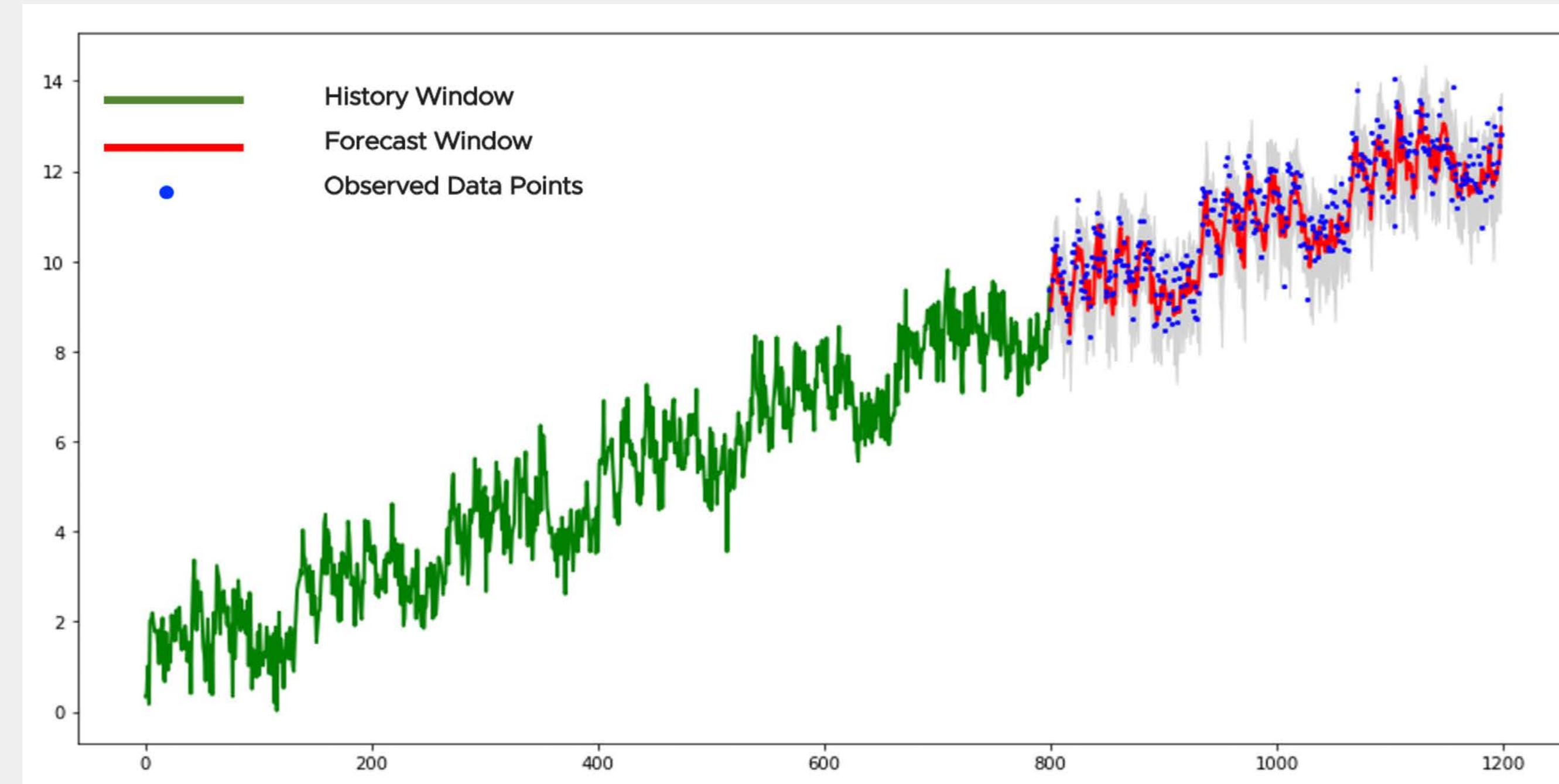
- The era of time-domain astronomy.
- Common analysis methods : gaussian processes, polynomial models, machine learning etc.



Large Synoptic Survey Telescope (LSST)



ARIMA models

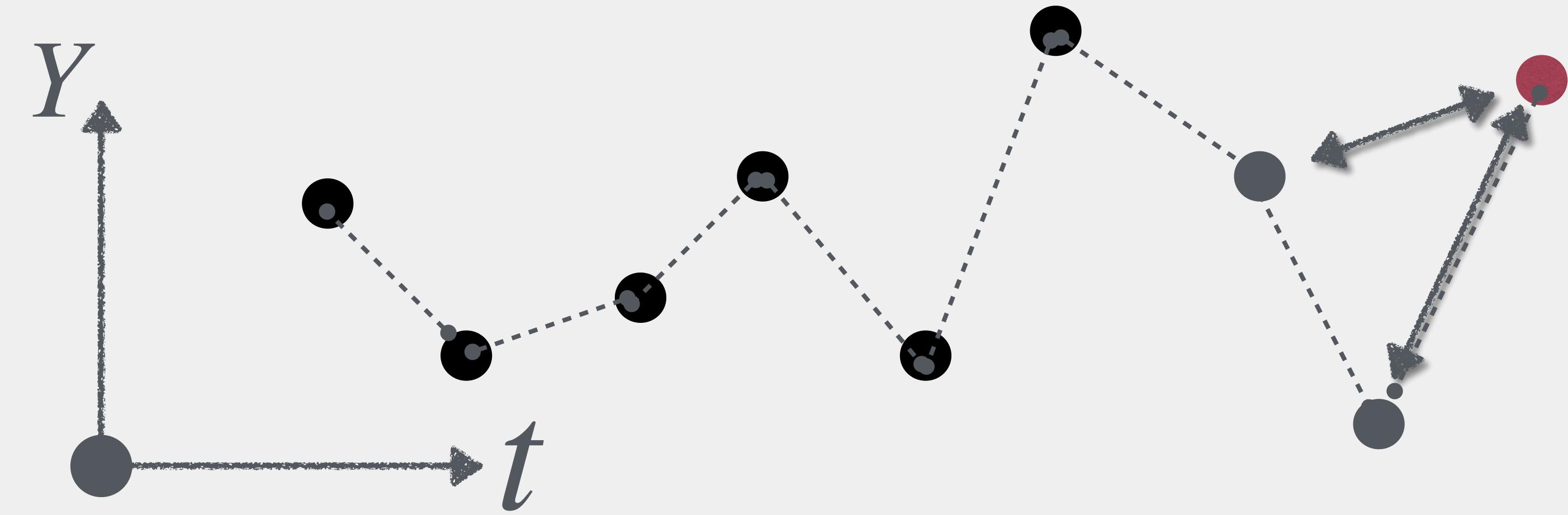


- Analysing and forecasting points \hat{y}_t for a given time series y_t
- **Autoregressive (AR), Integrated (I), Moving Average (MA)**

What are ARIMA Models?

Autoregressive AR(p) :

- Modelling “autocorrelation” in time series.
- Each datapoint correlated with its own previous (or “lagged”) values.
- For example : Daily average temperature



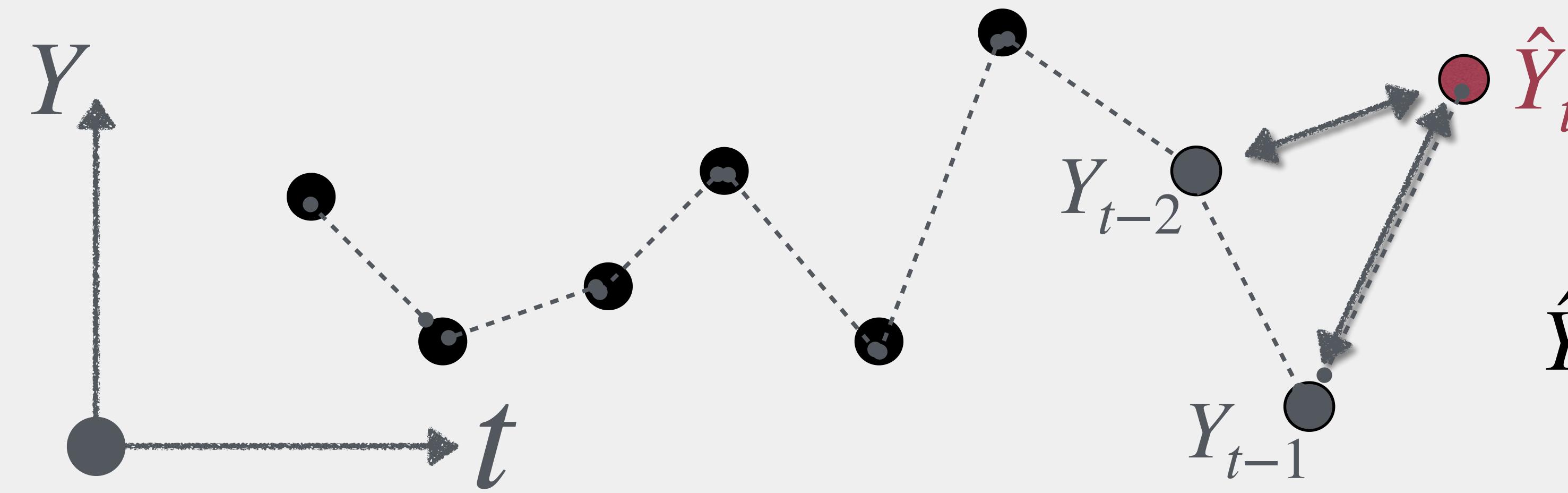
What are ARIMA Models?

Autoregressive AR(**p**) :

- Introduced in 1927, by Yule to model sunspot numbers.
- **p** denotes number of lagged terms.
- For example p=2 :



Udny Yule



$$\hat{Y}_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t$$

What are ARIMA Models?

Moving Average MA(q) :

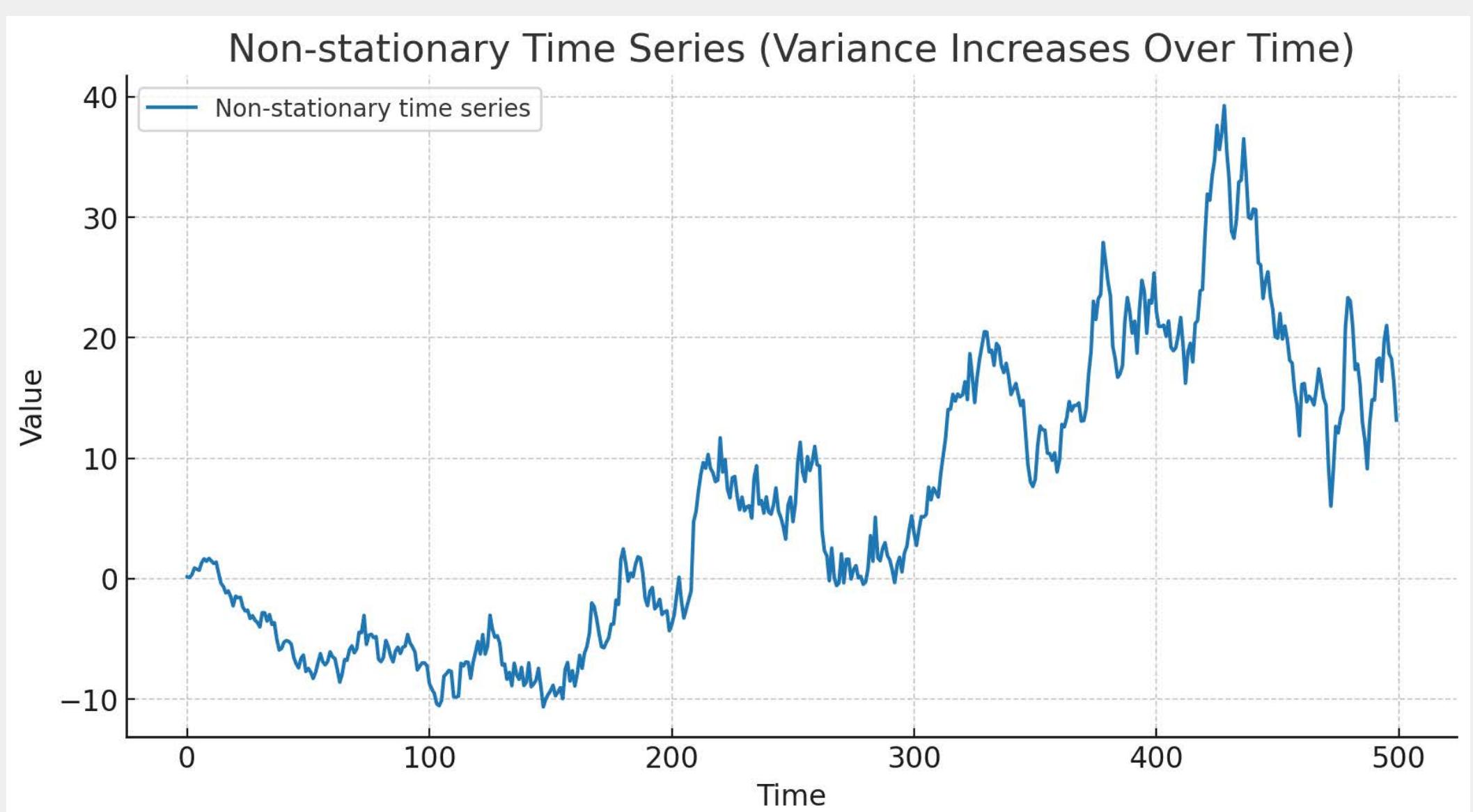
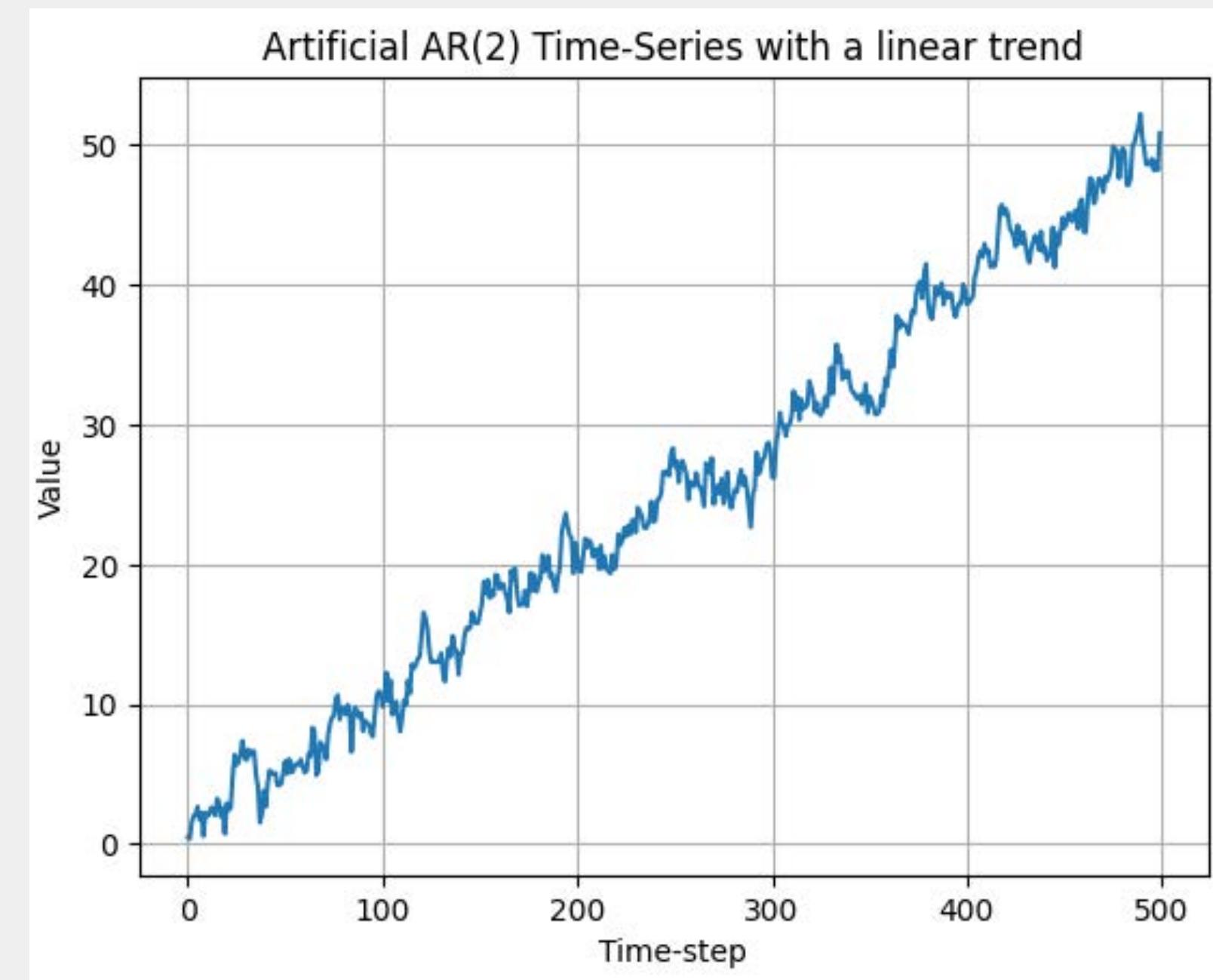
- Similar to **AR** models but present points correlated with lagged forecast errors (residuals).
- q—> number of lagged forecast errors. For example, MA(2) model :

$$\hat{y}_t = \mu + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \epsilon_t$$

What are ARIMA Models?

Integrated I (d) :

- ARMA modelling requires a stationary time series i.e. constant mean and variance.
- Integrated (I) part of ARIMA takes care of this by de-trending the time series using finite differencing.
- $d \rightarrow$ Order of differencing



What are ARIMA Models?

ARIMA (p, d, q):

- Combined into **ARIMA (p, d, q)** by Box and Jenkins in 1971.
- Used widely in economics, finance and weather/climate predictions.
- Not so common in Astronomy

$$\hat{y}_t = \mu + \phi_p y_{t-p} + \theta_q \epsilon_{t-q} + \epsilon_t$$

What are ARIMA Models?

ARIMA (p , d , q):

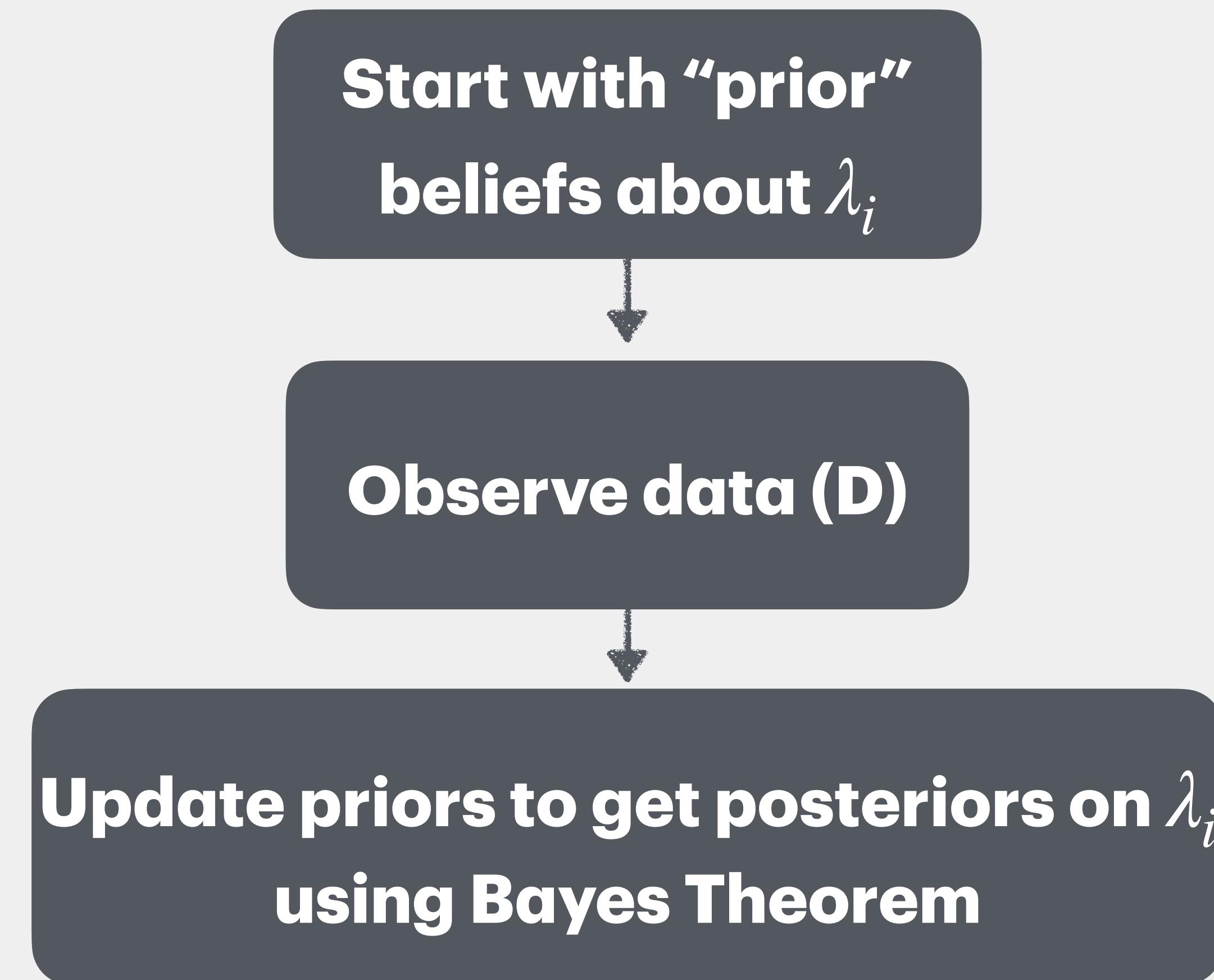
- p , d and q could be any positive integers.
- Difficult to select the right (p , d , q) order for fitting data.
- ARIMA models are over-parameterised, always a risk of overfitting.
- Need a method to choose the correct ARIMA Model for any given data.

$$\hat{y}_t = \mu + \phi_p y_{t-p} + \theta_q \epsilon_{t-q} + \epsilon_t$$

Bayesian Inference and Nested Sampling

A Primer on Bayesian Inference

- Infer the distribution of parameter values λ_i of a model **M** from data **D**.



Bayesian Inference and Nested Sampling

A Primer on Bayesian Inference :

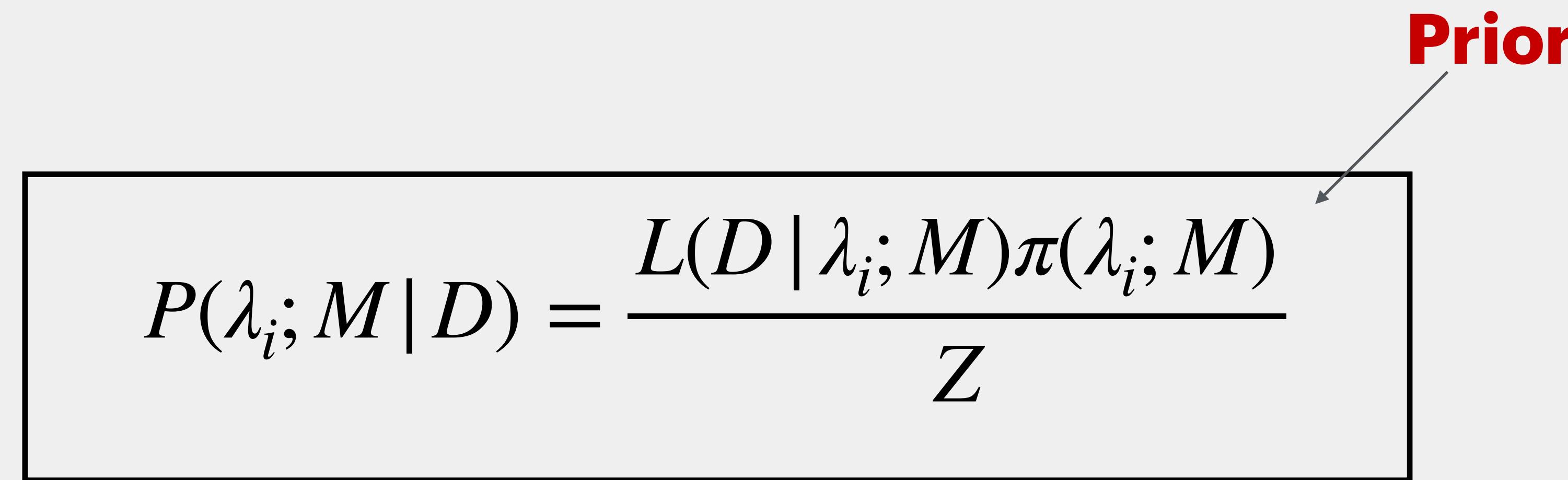
$$P(\lambda_i; M | D) = \frac{L(D | \lambda_i; M) \pi(\lambda_i; M)}{Z}$$

Bayesian Inference and Nested Sampling

A Primer on Bayesian Inference :

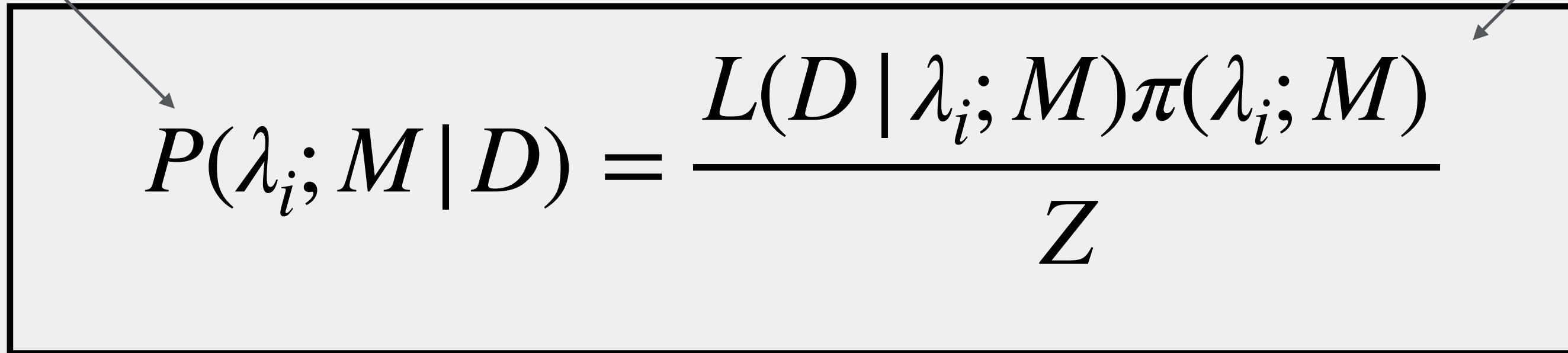
$$P(\lambda_i; M | D) = \frac{L(D | \lambda_i; M) \pi(\lambda_i; M)}{Z}$$

Prior



Bayesian Inference and Nested Sampling

A Primer on Bayesian Inference :



The diagram shows the Bayesian posterior formula $P(\lambda_i; M | D) = \frac{L(D | \lambda_i; M)\pi(\lambda_i; M)}{Z}$ enclosed in a black rectangular box. A green arrow points from the text "Posterior" to the left side of the box. A red arrow points from the text "Prior" to the right side of the box.

$$P(\lambda_i; M | D) = \frac{L(D | \lambda_i; M)\pi(\lambda_i; M)}{Z}$$

Bayesian Inference and Nested Sampling

A Primer on Bayesian Inference :

$$P(\lambda_i; M | D) = \frac{L(D | \lambda_i; M) \pi(\lambda_i; M)}{Z}$$

Diagram illustrating the Bayesian formula:

- Posterior** (green text) is associated with the term $P(\lambda_i; M | D)$.
- Likelihood** (blue text) is associated with the term $L(D | \lambda_i; M)$.
- Prior** (red text) is associated with the term $\pi(\lambda_i; M)$.
- A vertical arrow points from the Likelihood term to the formula.
- Two arrows point from the Prior term to the formula: one from the right side and one from the bottom right corner.

Bayesian Inference and Nested Sampling

A Primer on Bayesian Inference :

$$P(\lambda_i; M | D) = \frac{L(D | \lambda_i; M) \pi(\lambda_i; M)}{Z}$$
$$Z = \int L(D | \lambda_i) \pi(\lambda_i) d\lambda_i$$

Posterior **Likelihood** **Prior**

Bayesian Inference and Nested Sampling

A Primer on Bayesian Inference :

$$P(\lambda_i; M | D) = \frac{L(D | \lambda_i; M) \pi(\lambda_i; M)}{Z}$$

Diagram illustrating the Bayesian formula:

- Posterior** (green) is labeled on the left, with an arrow pointing to the term $P(\lambda_i; M | D)$.
- Likelihood** (blue) is labeled above the formula, with an arrow pointing to the term $L(D | \lambda_i; M)$.
- Prior** (red) is labeled on the right, with an arrow pointing to the term $\pi(\lambda_i; M)$.
- Evidence** (purple) is labeled below the formula, with an arrow pointing to the term Z .

$$\text{Evidence : } Z = \int L(D | \lambda_i) \pi(\lambda_i) d\lambda_i$$

Bayesian Inference and Nested Sampling

A Primer on Bayesian Inference :

$$P(\lambda_i; M | D) = \frac{L(D | \lambda_i; M) \pi(\lambda_i; M)}{Z}$$

Posterior **Likelihood** **Prior**

Evidence : $Z = \int L(D | \lambda_i) \pi(\lambda_i) d\lambda_i$

Useful for model comparison!!

The diagram illustrates the Bayesian posterior formula. A large rectangular box contains the equation $P(\lambda_i; M | D) = \frac{L(D | \lambda_i; M) \pi(\lambda_i; M)}{Z}$. Four arrows point to different parts of the equation: an arrow from the text 'Posterior' points to the term $P(\lambda_i; M | D)$; an arrow from the text 'Likelihood' points to the term $L(D | \lambda_i; M)$; an arrow from the text 'Prior' points to the term $\pi(\lambda_i; M)$; and an arrow from the text 'Evidence' points to the denominator Z .

Bayesian Inference and Nested Sampling

Evidence for Model Comparison

$$Z_n = \int L(D | \lambda_i; M_n) \pi(\lambda_i; M_n) d\lambda_i = P(\mathbf{D} | \mathbf{M}_n)$$

- Model with higher evidence statistically preferred by data.
- But, cumbersome to evaluate due to “curse of dimensionality”.
- Solution → Nested Sampling!

Bayesian Inference and Nested Sampling

The Nested Sampling Algorithm

- Introduced by physicist John Skilling in 2003.
- Key idea is to define the “prior volume” - amount of prior mass contained inside an equal likelihood contour.

$$X(L) = \int_{L>L(\lambda)} \pi(\lambda) d\lambda$$

- Transform the multi-dimensional evidence integral to a simple one-dimensional integral:

$$Z(X) = \int_0^1 L(X) dX$$

ARIMA x Nested Sampling

The Idea

$$\hat{y}_t = \mu + \phi_p y_{t-p} + \theta_q \epsilon_{t-q} + \epsilon_t$$

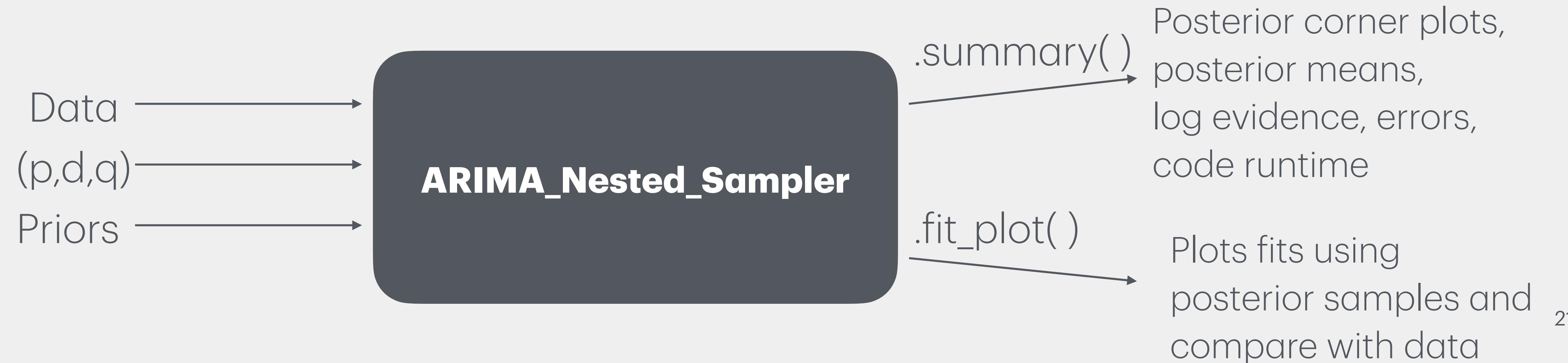
$$P(\lambda_i; M | D) = \frac{L(D | \lambda_i; M) \pi(\lambda_i; M)}{Z}$$

- Use the weights (ϕ_p, θ_q) and the standard deviation σ characterising ϵ_t as parameters λ_i for Bayesian Inference.
- Nested Sampling serves as an efficient tool : model selection + posterior distributions for parameters.
- Occam's penalty ensures overfitting is avoided.

ARIMA x Nested Sampling

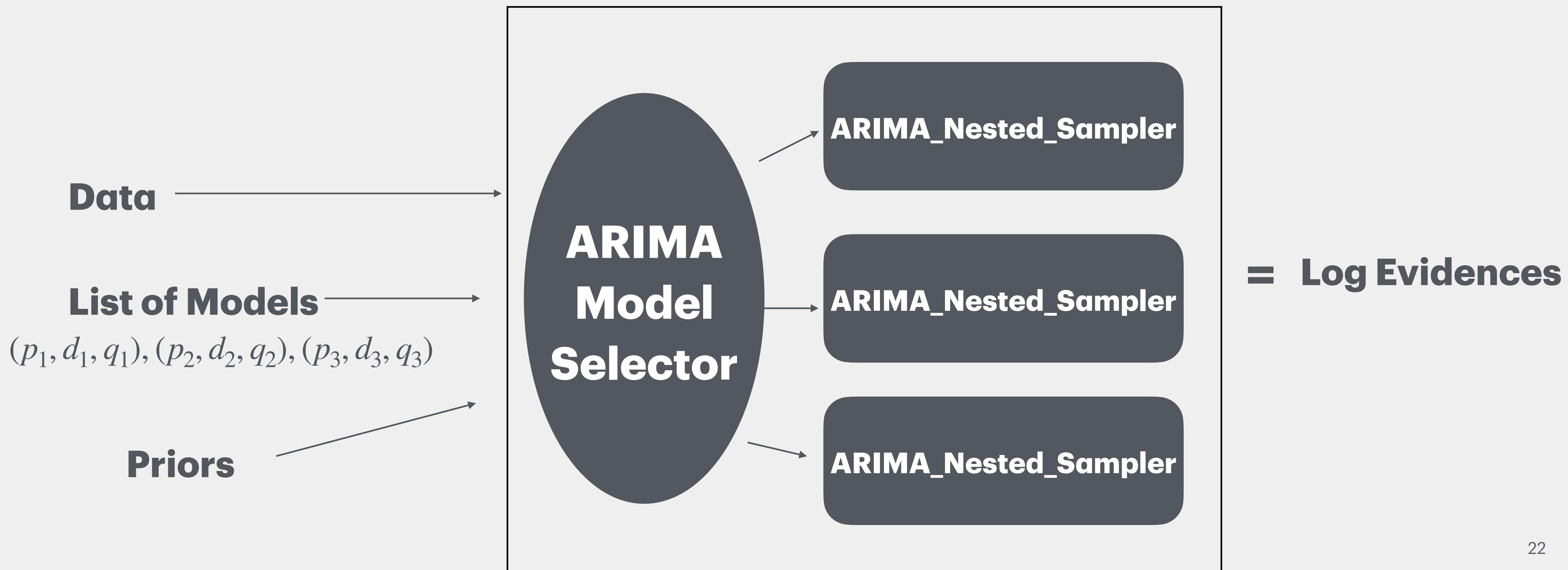
The Code

- **BlackJAX** Nested Sampler.
- Leveraging the JAX ecosystem (runtime reduced from 3-4 minutes to just few seconds!)
- Main object : **ARIMA_Nested_Sampler** class



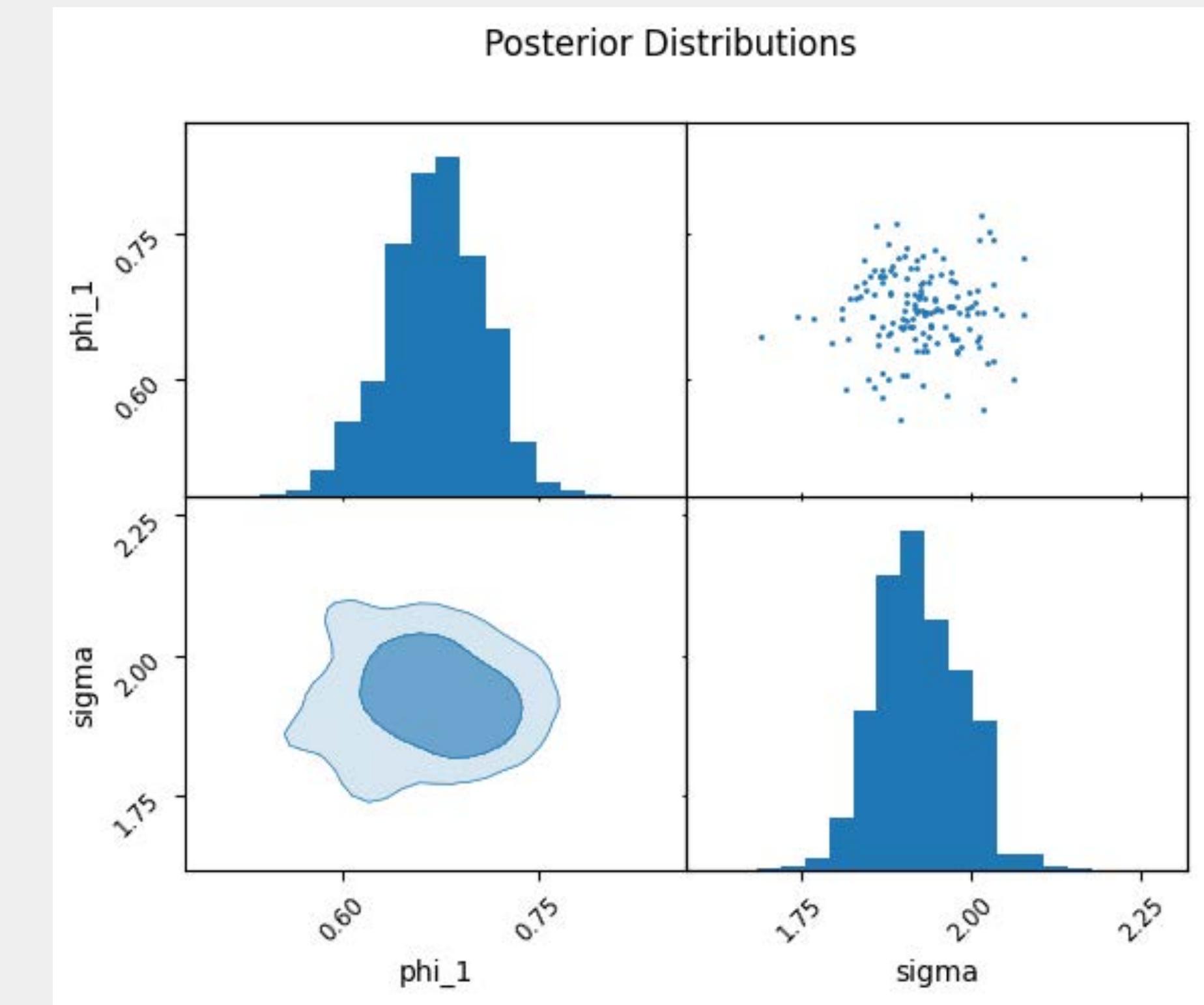
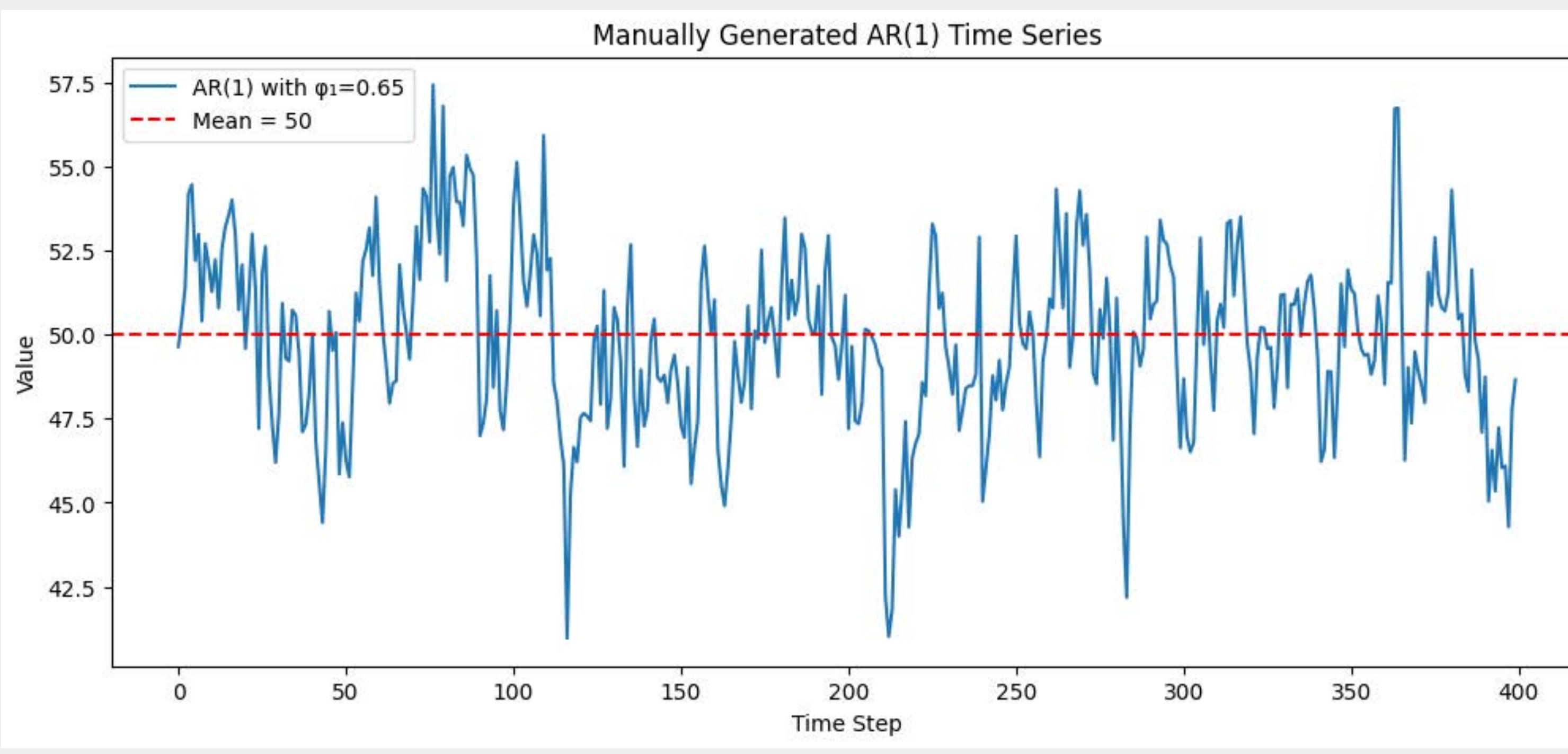
ARIMA x Nested Sampling

Model Comparison



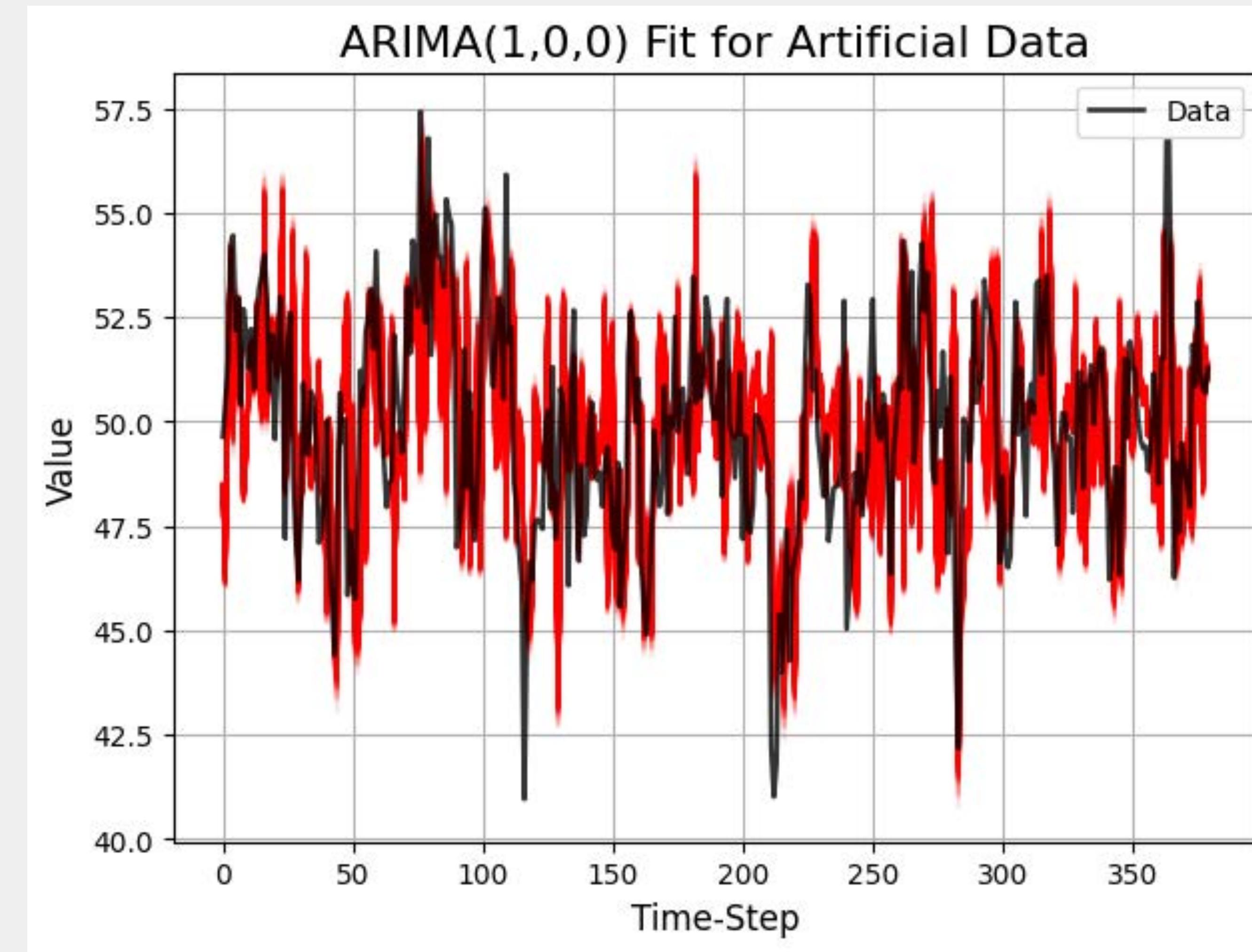
ARIMA x Nested Sampling

Testing on synthetic data



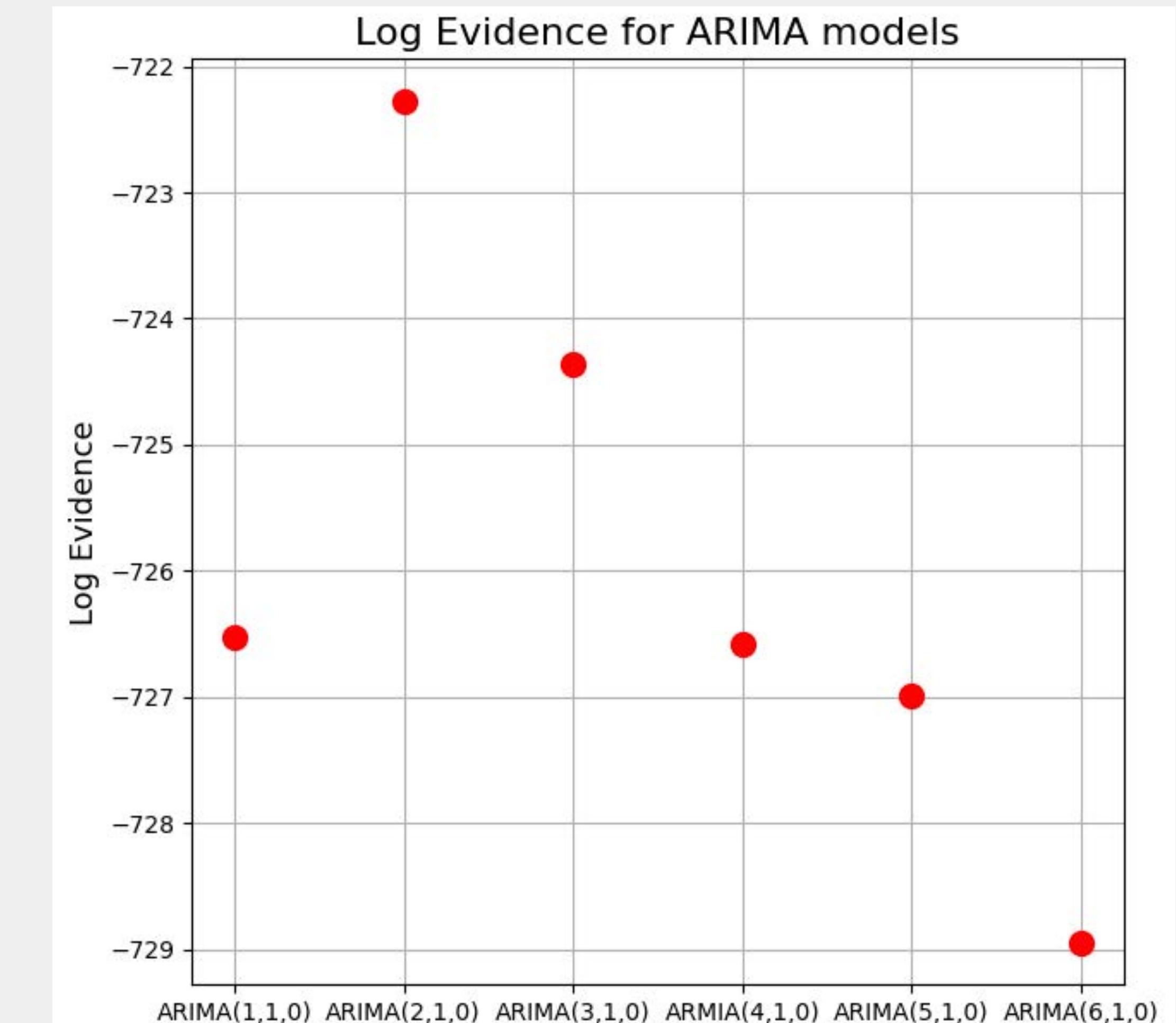
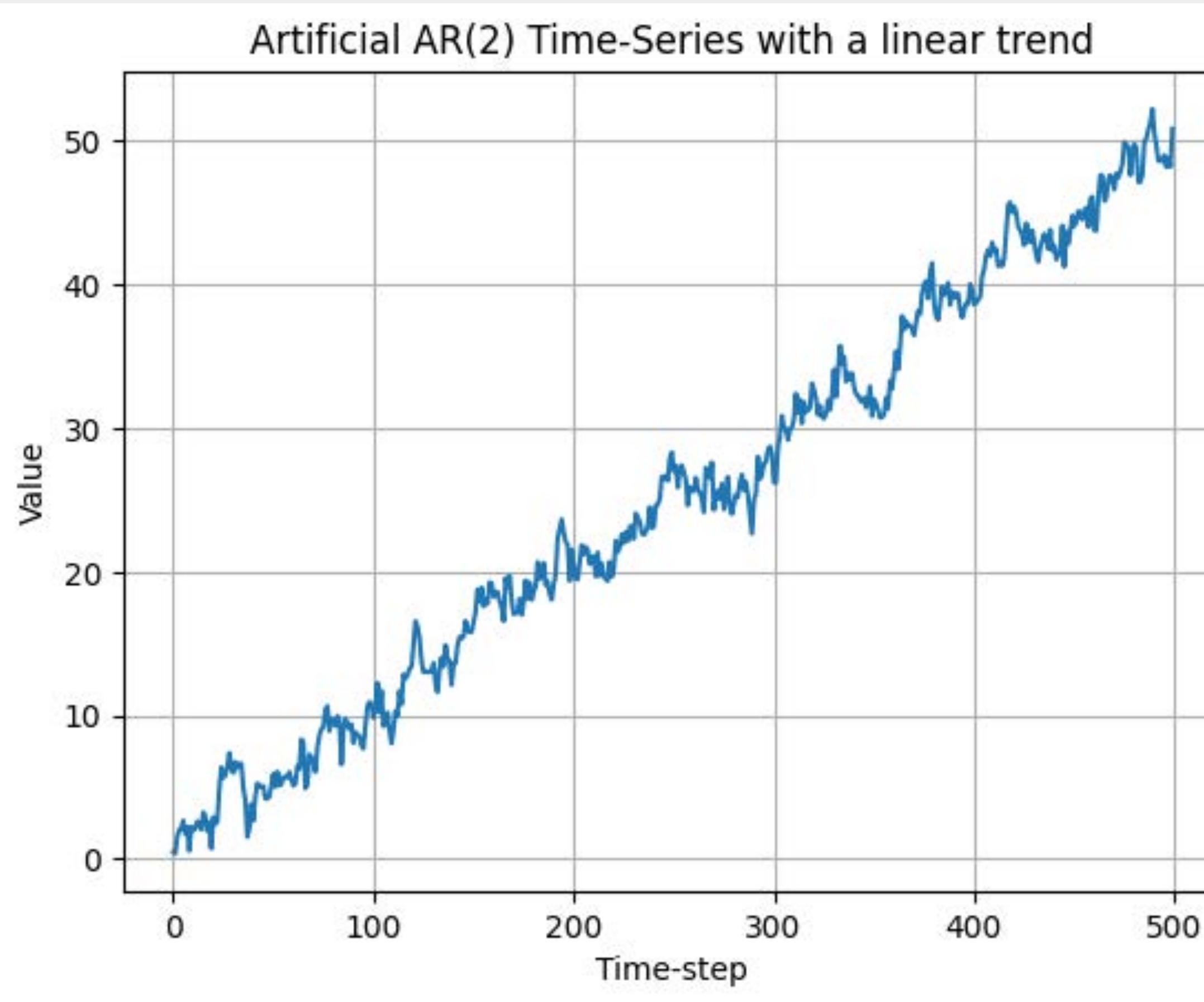
ARIMA x Nested Sampling

Testing on synthetic data



ARIMA x Nested Sampling

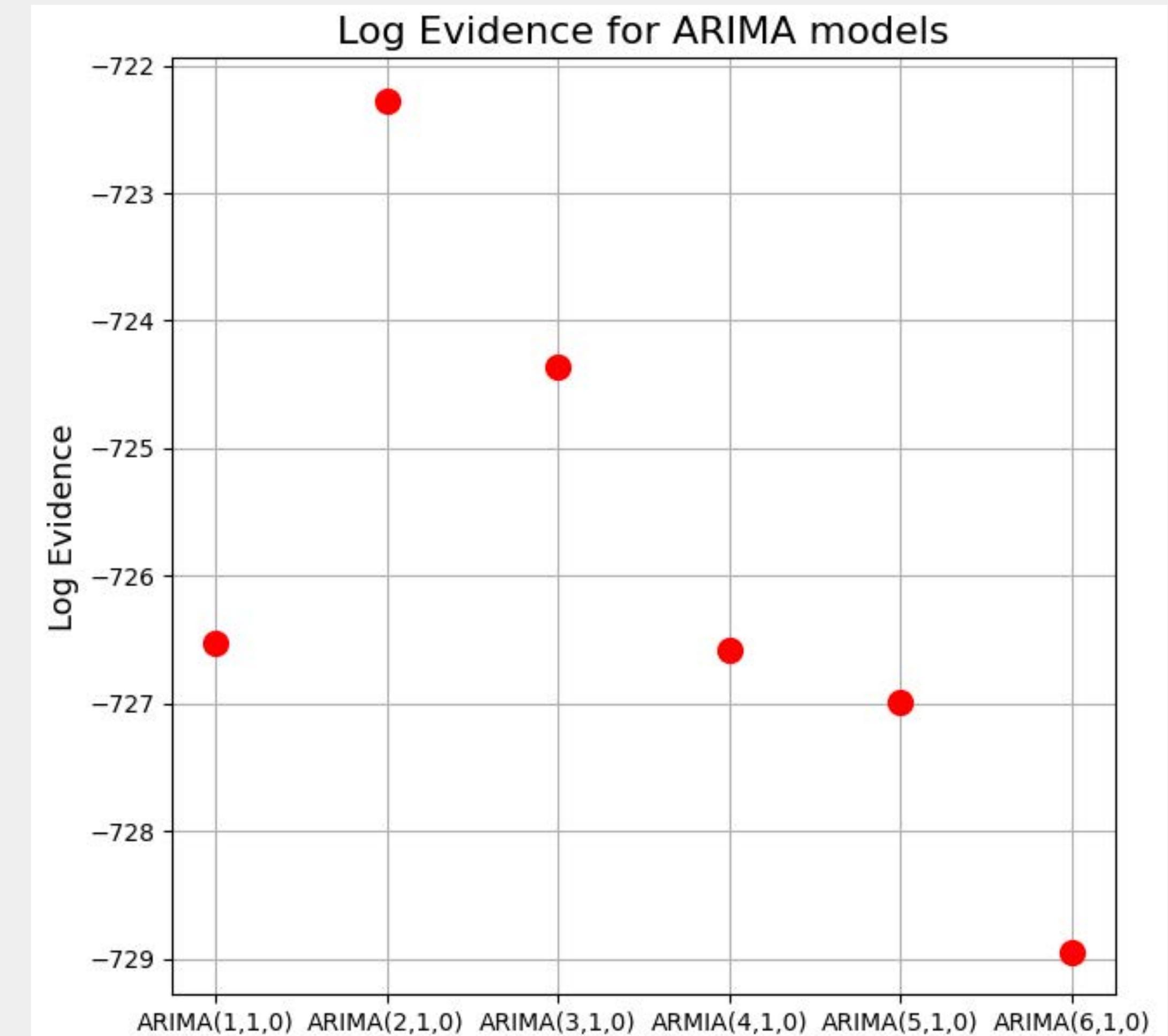
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ARIMA x Nested Sampling

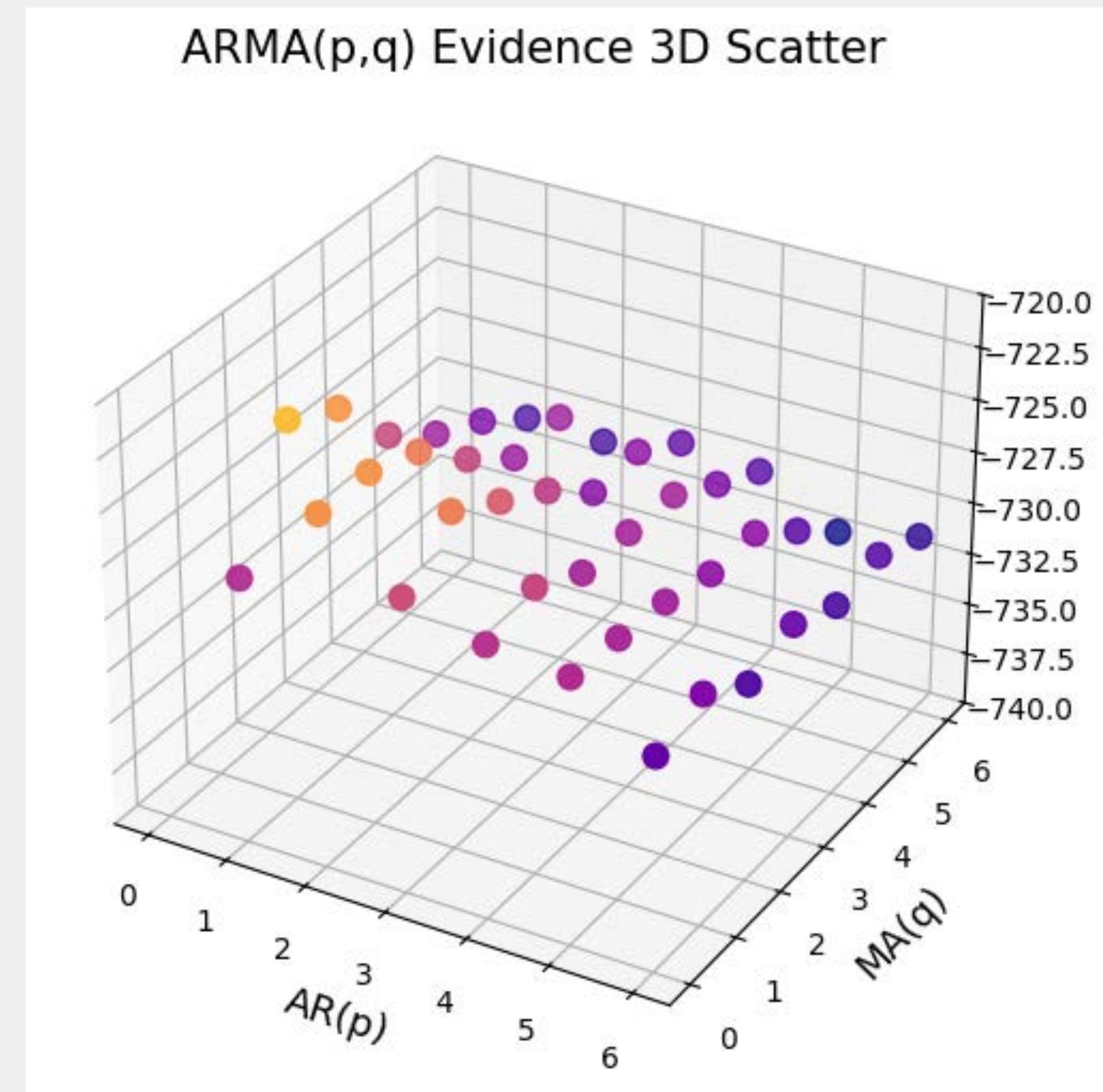
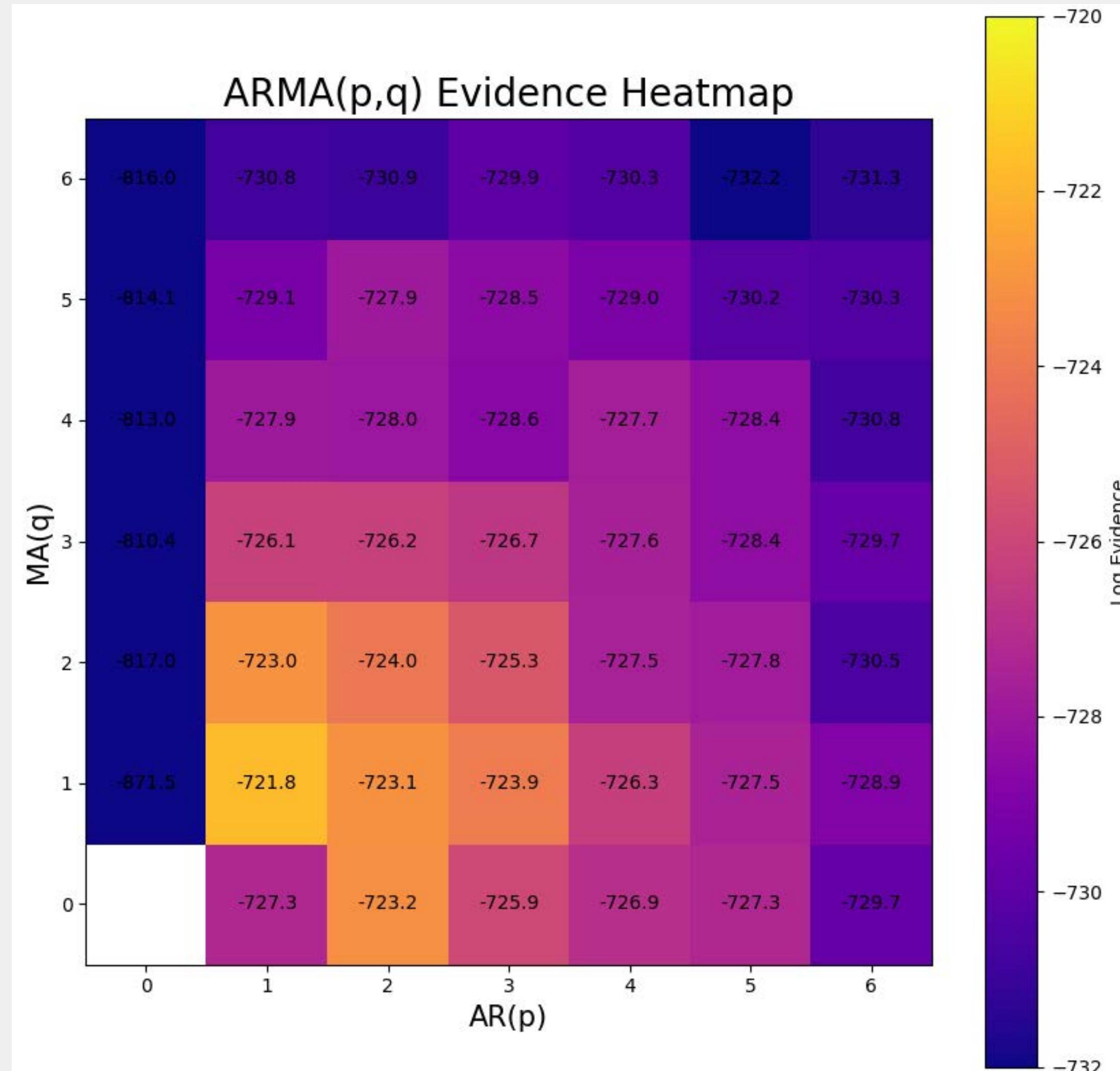
The Occam's Penalty in Action

$$Z = \int L(\theta | D) \pi(\theta) d\theta$$



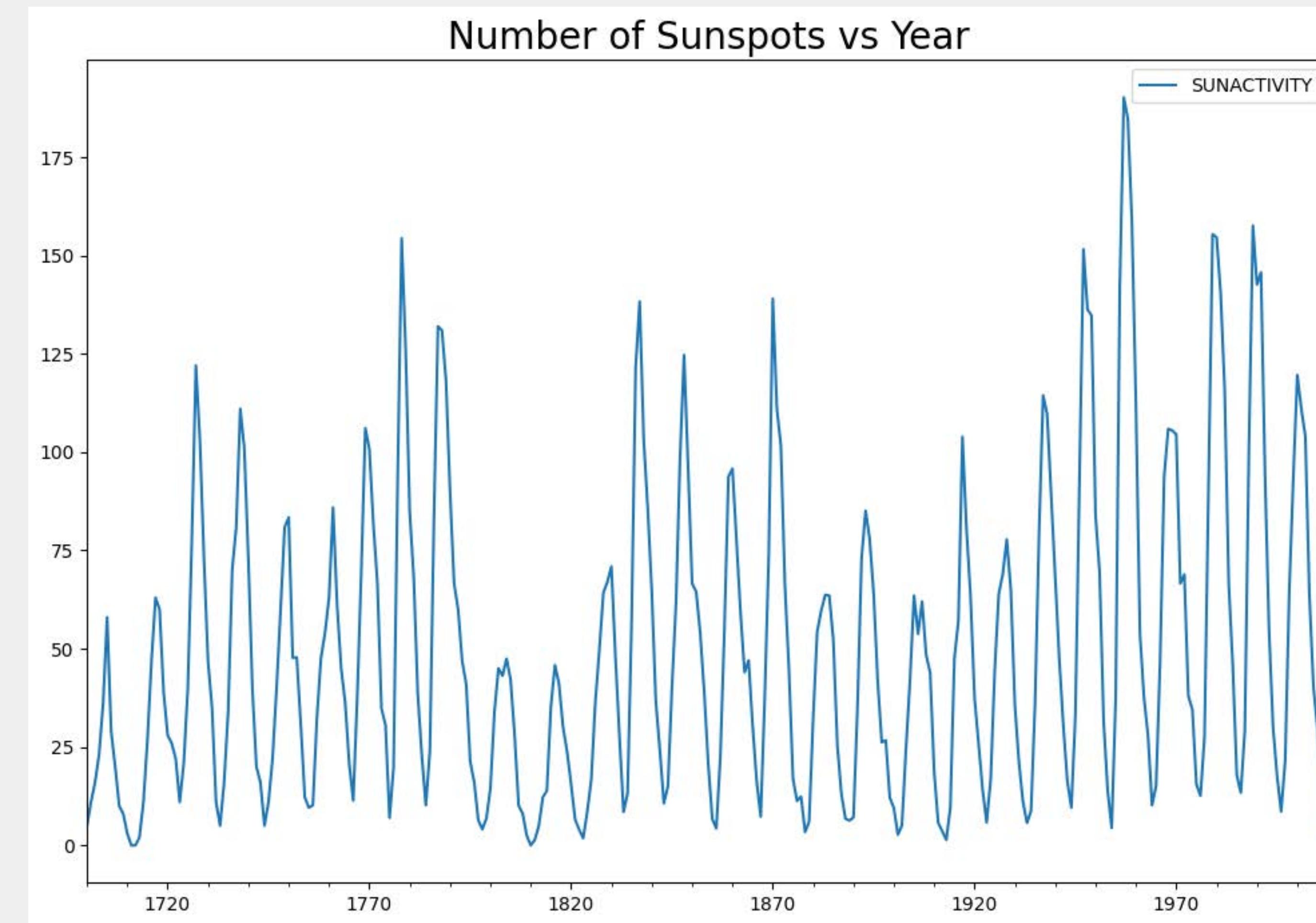
ARIMA x Nested Sampling

Testing on synthetic data



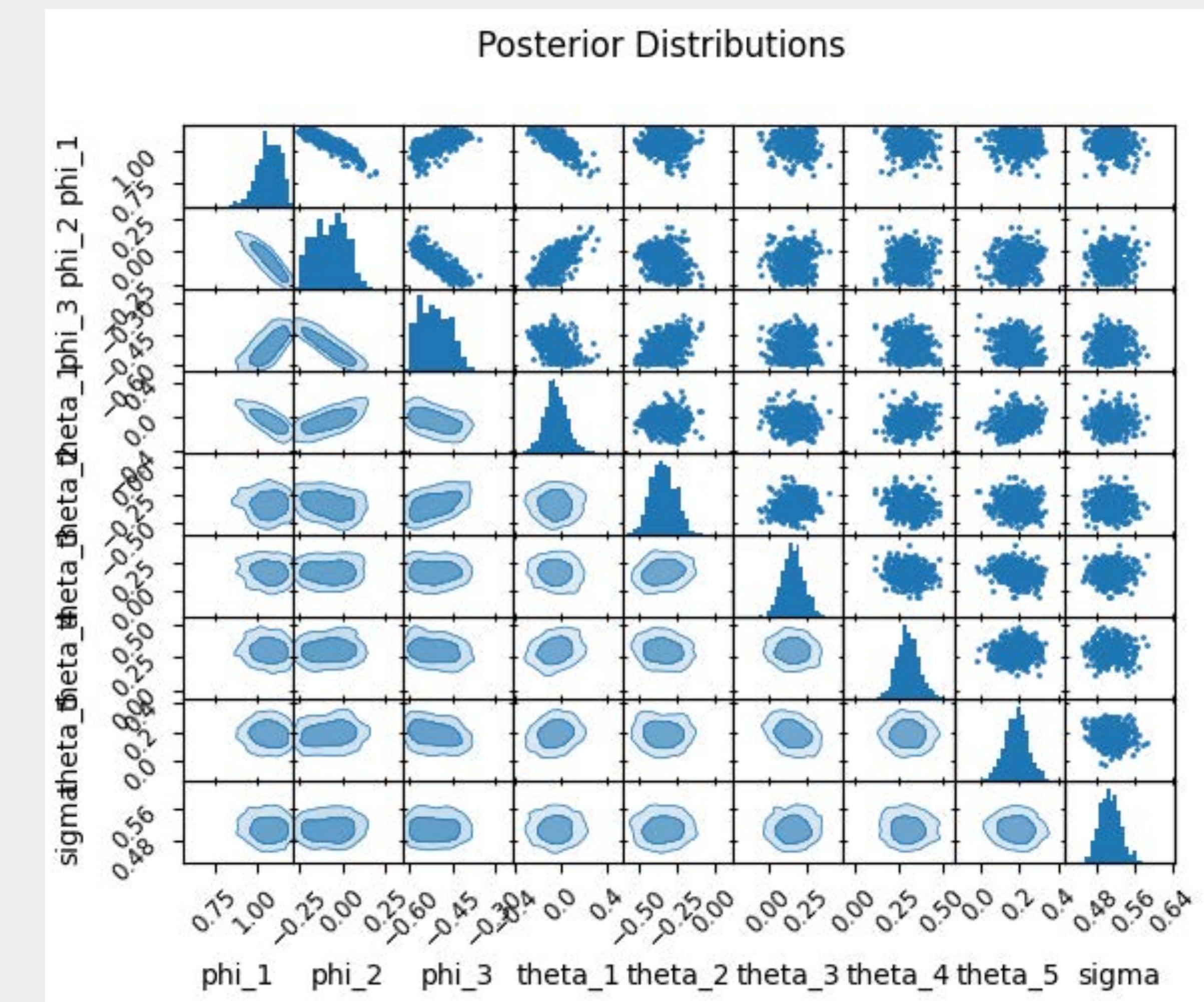
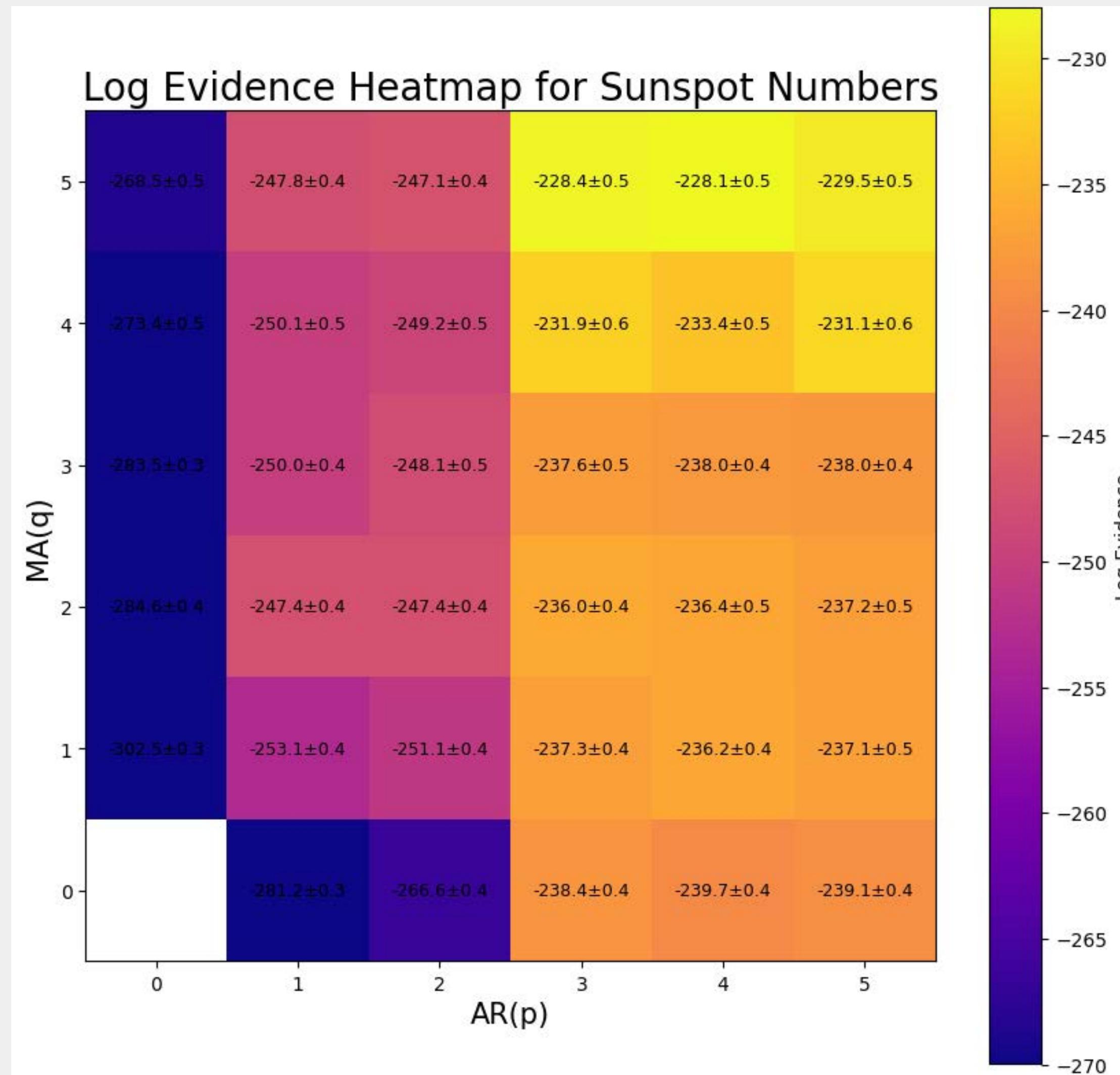
Astronomical Case Study

Sunspot Numbers



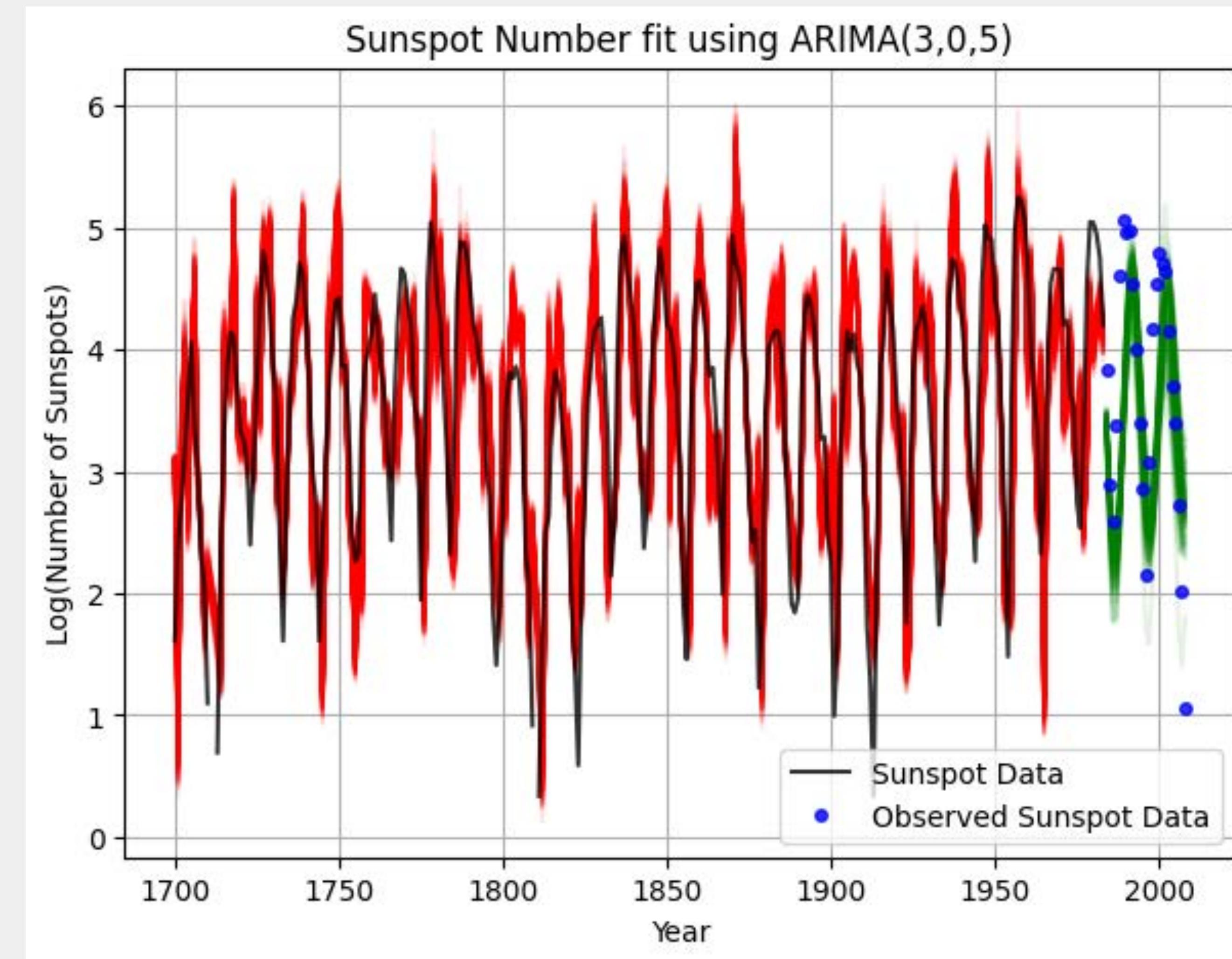
Astronomical Case Study

Sunspot Numbers



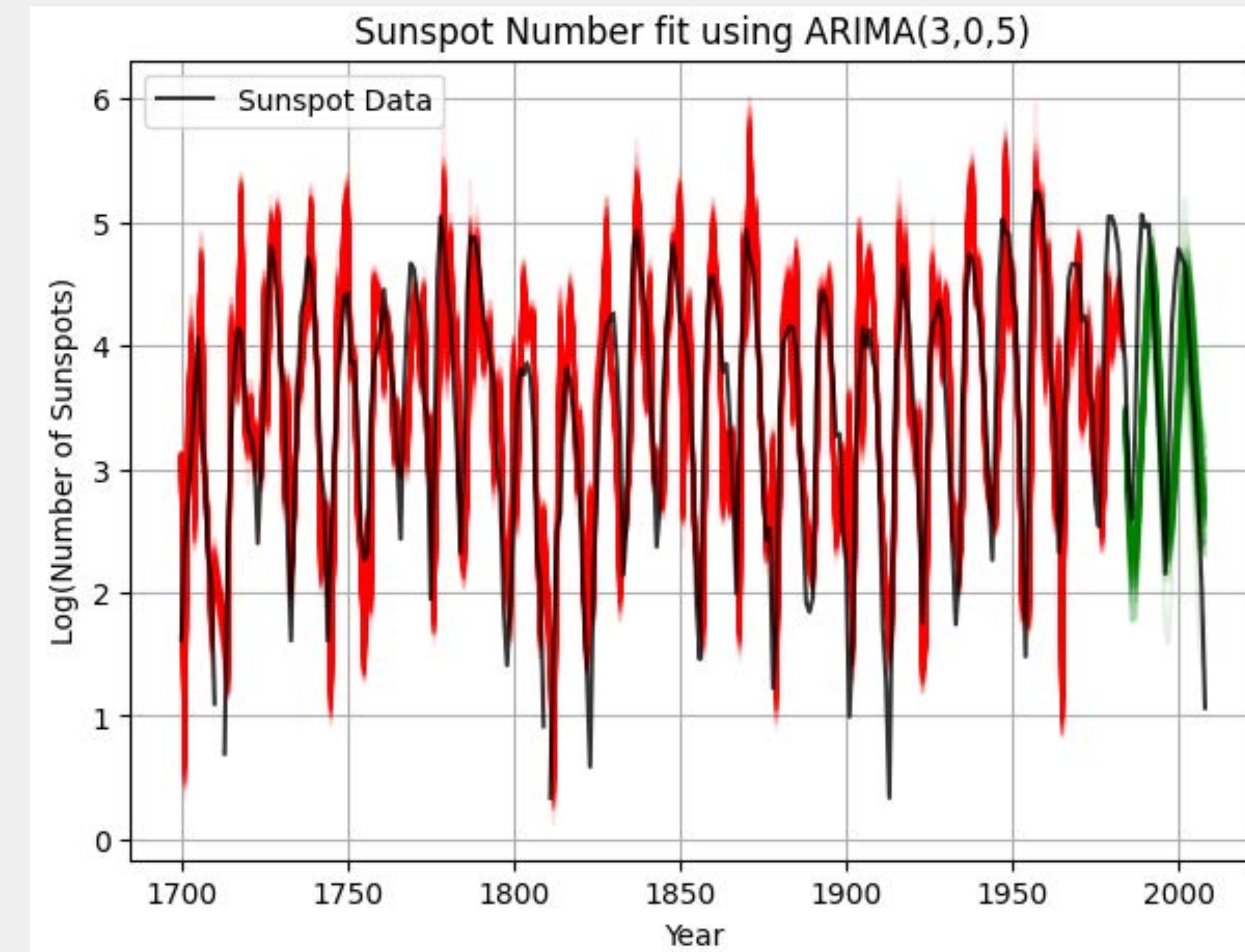
Astronomical Case Study

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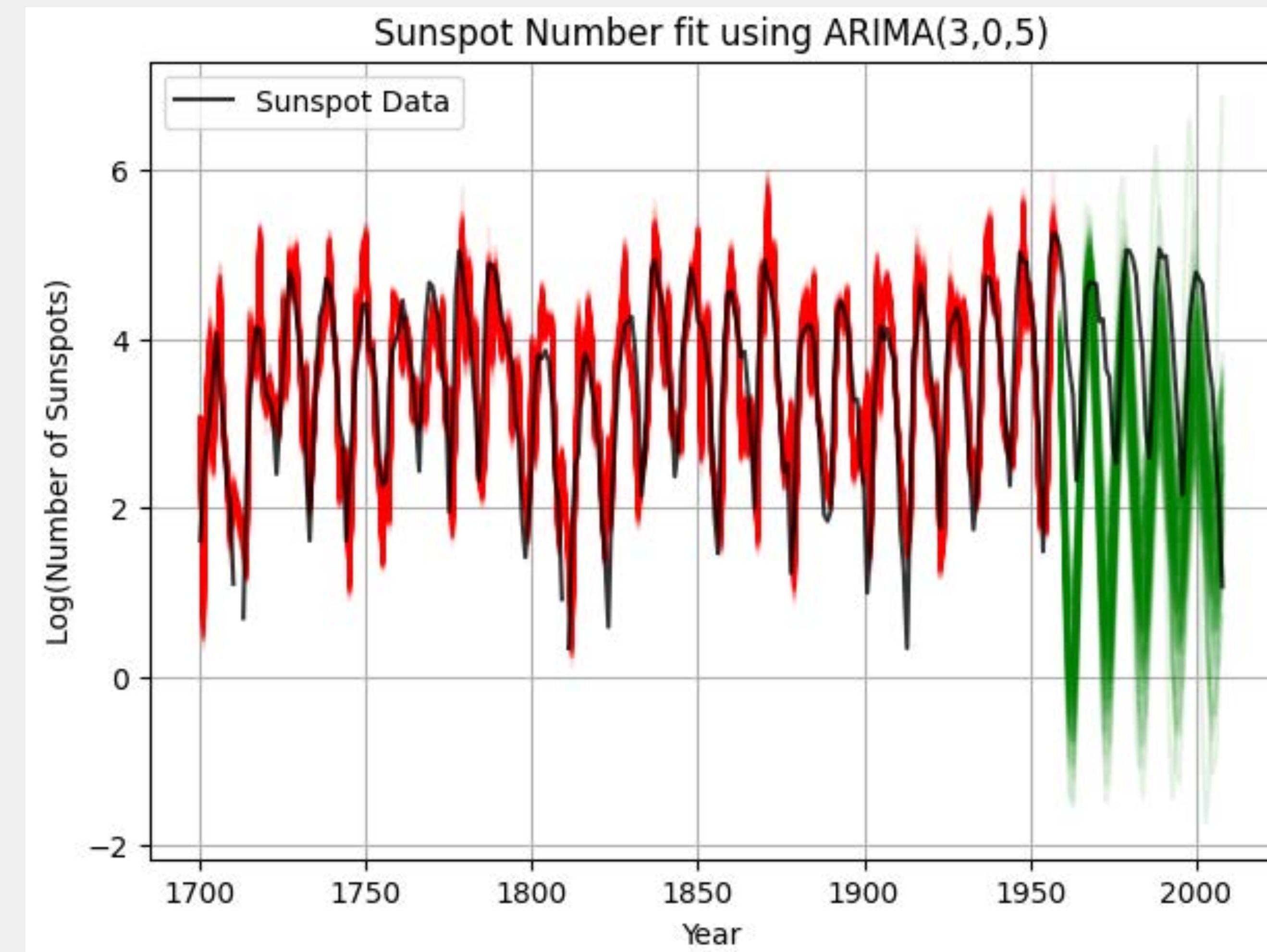
Astronomical Case Study

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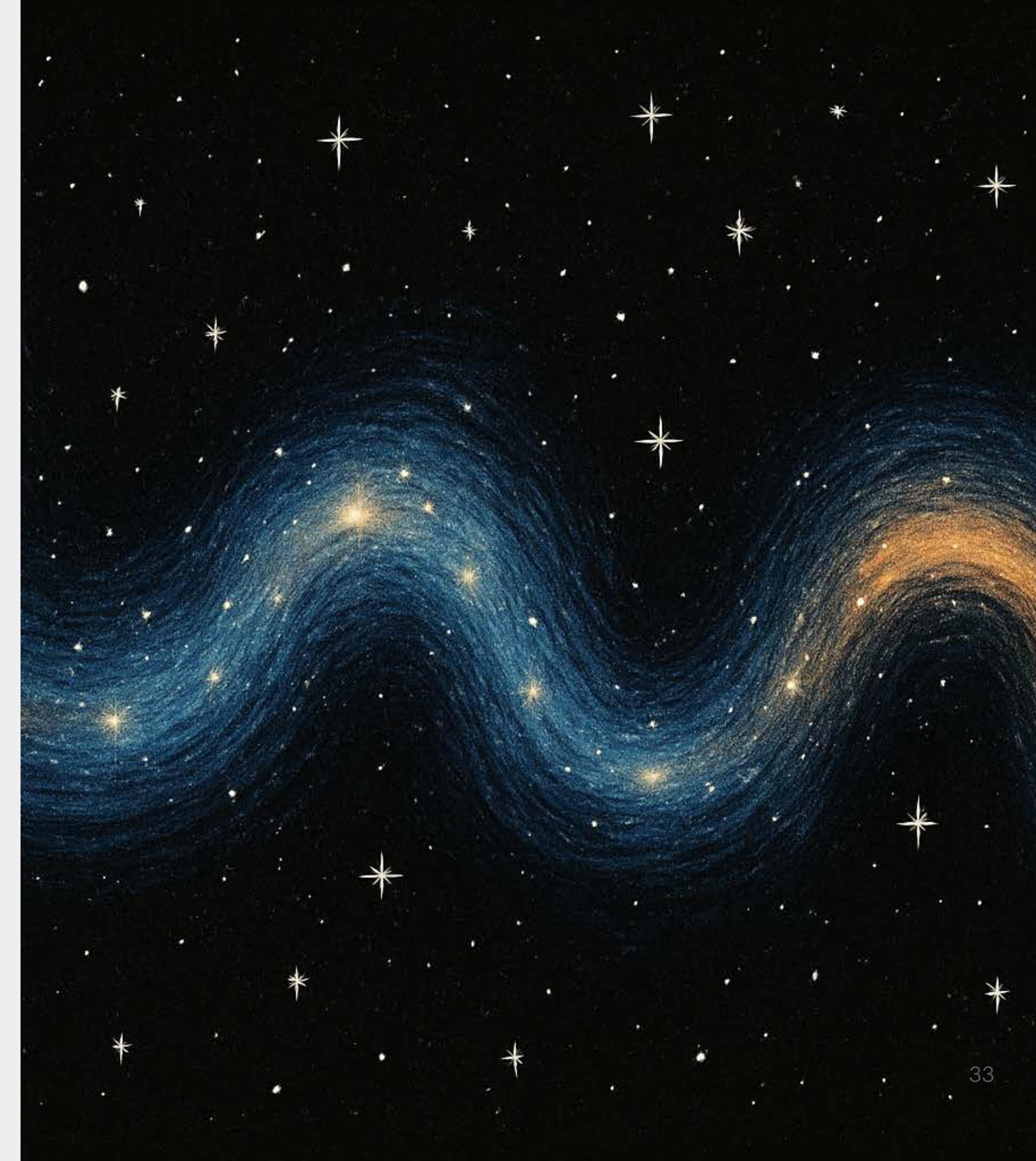
Astronomical Case Study

Sunspot Numbers



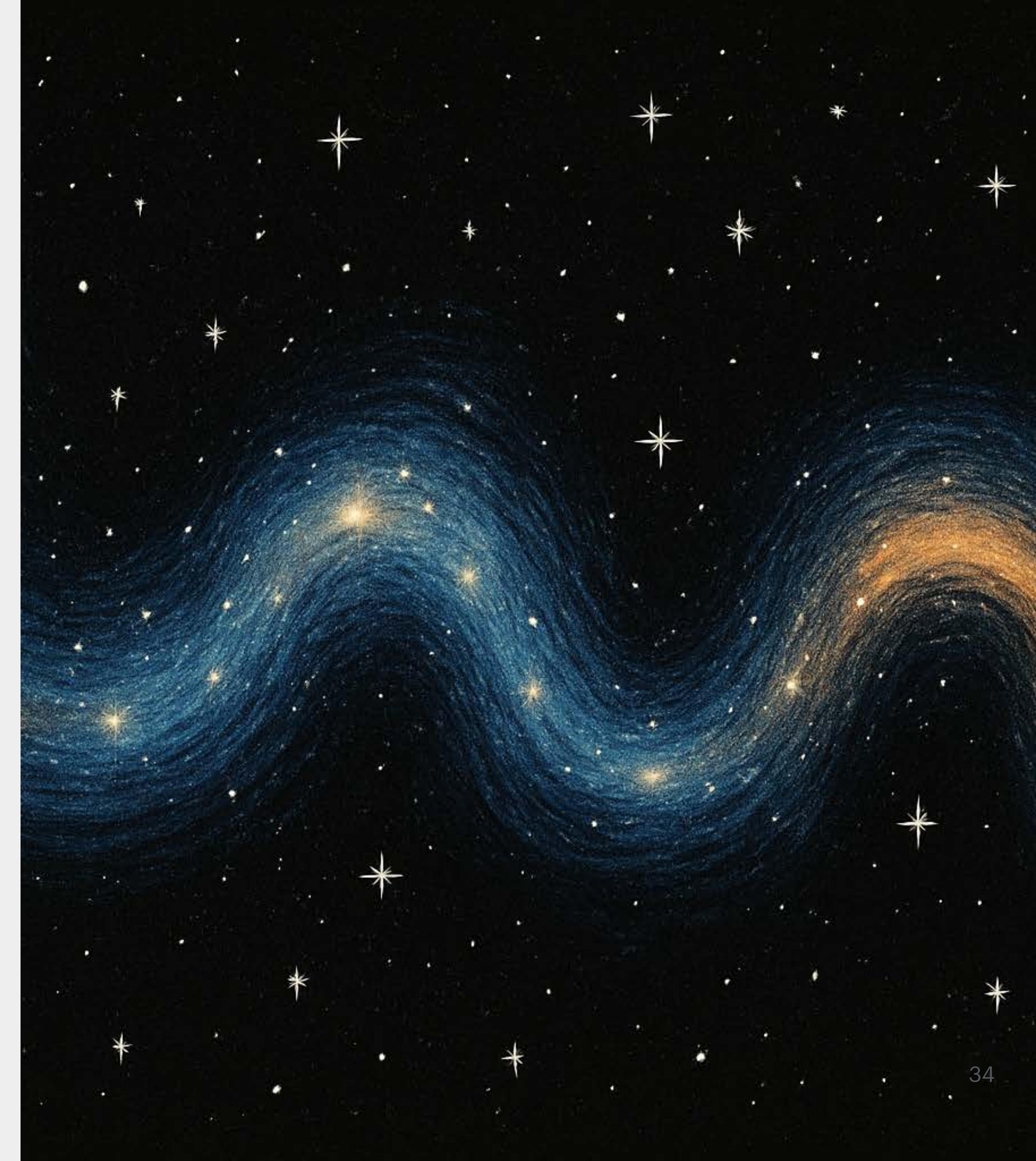
Limitations

- Requires data which is evenly spaced in time.
- Cannot capture long-term, seasonal trends.
- Lack of physical interpretations



Future Prospects

- Extending to other hybrid ARIMA models :Seasonal ARIMA, Continuous ARIMA, and so on.
- Implement on more datasets : AGN and quasar light curves, residual analysis, noise characterisation for gravitational wave data.
- Categorise astronomical datasets on the basis of preferred ARIMA models
—> possible physical insights?





Thank You!