Cosmic strings and boson stars

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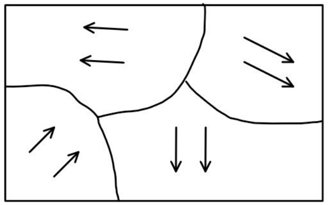
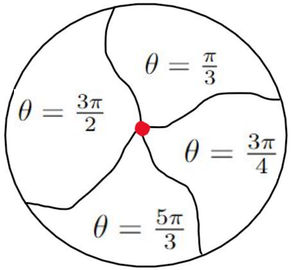
What is a cosmic string?

- Type of 1-dimensional topological defect
  - Symmetry broken due to phase transition
  - Simplest case: U(1) symmetry breaking by a complex scalar field
- Gravitational effects:
  - Deficit angle
  - Can emit gravitational waves
- Massless and massive modes of radiation

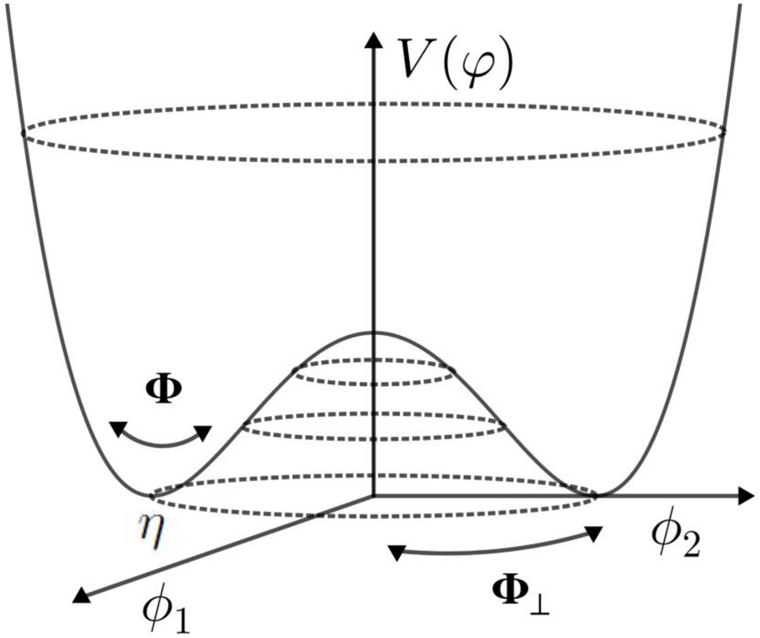


Ferromagnet



Cosmic string

Potential of a cosmic string: $V(\varphi) = \frac{1}{4}\lambda\left(\bar{\varphi}\varphi - \eta^2\right)^2$



Lagrangian: $\mathcal{L} = (\partial_\mu\bar{\varphi})(\partial^\mu\varphi) - V(\varphi)$

Invariant under $\varphi(x) \to e^{i\alpha}\varphi(x)$ symmetry transform

What is a boson star?

- Boson field condenses into gravitationally bound compact object
  - Supported by self-gravity and/or scalar self-interaction
  - No singularities or horizon (unlike black holes)
- Complex scalar field
- Properties:
  - Can be very compact (like neutron stars)
  - Could detect by gravitational radiation

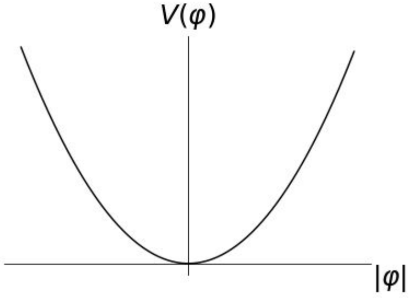
Types of boson-star potential

Boson star:

$$V = \mu^2|\varphi|^2$$

$$V = \mu^2|\varphi|^2 + \frac{\lambda}{2}|\varphi|^4$$

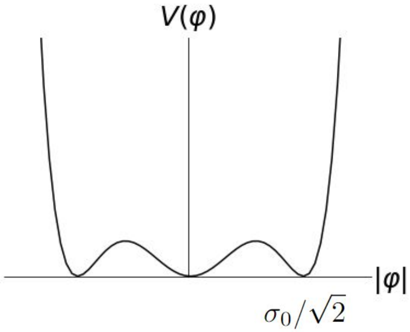
$$V_{sol} = \mu^2|\varphi|^2\left(1 - \frac{2|\varphi|^2}{\sigma_0^2}\right)^2$$



Boson star

Another interesting object: Q-balls - solitonic boson star without gravity

- Non-gravitational soliton
- Q: conserved charge



Solitonic boson star

Initial profiles

For both CS and BS, vary EL to get (for their respective potentials)

$$\frac{\partial^2 \bar{\varphi}}{\partial t^2} - \nabla^2 \bar{\varphi} + \frac{\partial V}{\partial \bar{\varphi}} = 0$$

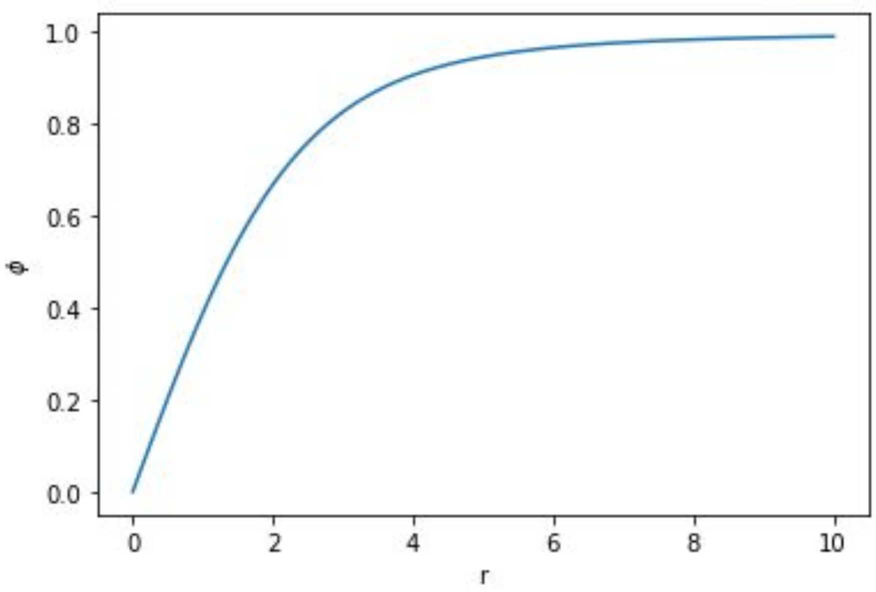
Solve CS: cylindrical ansatz $\varphi(r, \theta) = \phi(r) e^{i n_w \theta}$

Solve BS: spherical ansatz $\varphi(t, r) = \phi(r) e^{i \omega t}$

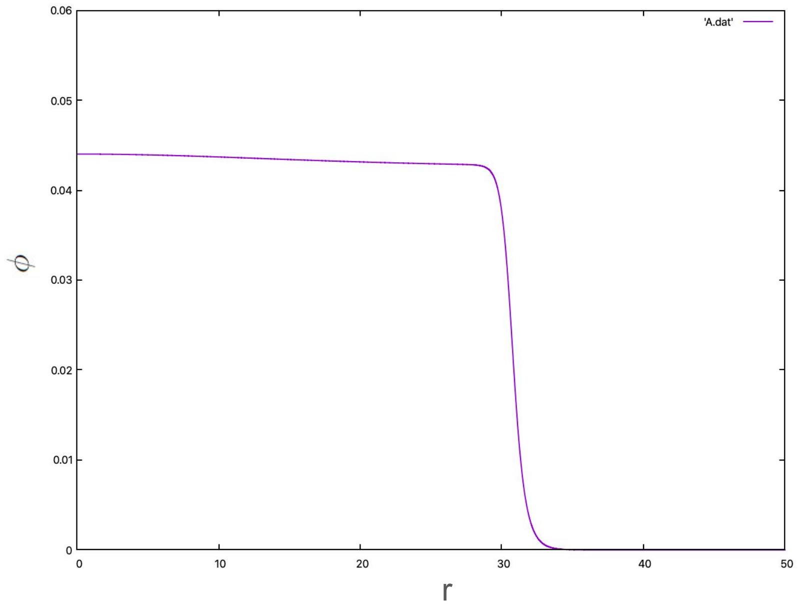
- Time-periodic field

Initial profiles

Cosmic string



Boson star



Boundary conditions:

$$\phi \to 0 \text{ at } r = 0,\ \phi \to \text{const. as } r \to \infty$$

$$\phi \text{ const. at } r = 0,\ \phi \to 0 \text{ as } r \to \infty$$

Radiation

Consider non-relativistic case.

Same process for cosmic strings and boson stars but with different potentials:

- General form: $\varphi(x^\mu) = \phi(x^\mu) e^{i\vartheta(x^\mu)}$
- Substitute into Euler-Lagrange equations
- Separate into real / imaginary parts
- Perturbation analysis: vary about vacuum state

Separation into real and imaginary parts

Real part of Euler-Lagrange equations (real EL):

$$\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \varphi = \phi \left[\left(\frac{\partial \vartheta}{\partial t} \right)^2 - (\nabla \vartheta)^2 - \frac{\partial V}{\partial \bar{\phi}} \right]$$

Imaginary part of Euler-Lagrange equations (imaginary EL):

$$\frac{\partial^2 \vartheta}{\partial t^2} - \nabla^2 \vartheta = \frac{2}{\phi} \left(\frac{\partial \phi}{\partial t} \frac{\partial \vartheta}{\partial t} - \nabla \phi \nabla \vartheta \right)$$

Analysis of imaginary EL

Imaginary part of Euler-Lagrange equations (imaginary EL):

$$\frac{\partial^2\vartheta}{\partial t^2} - \nabla^2\vartheta = \frac{2}{\phi}\left(\frac{\partial\phi}{\partial t}\frac{\partial\vartheta}{\partial t} - \nabla\phi\nabla\vartheta\right)$$

- Cosmic strings and solitonic boson stars: ϕ constant so

$$\frac{\partial^2\vartheta}{\partial t^2} - \nabla^2\vartheta = 0$$

- Boson stars: trivially satisfied by $\theta = \omega t$

Analysis of real EL

Substitute in the appropriate potentials:

Type	Potential V	Real part of EL
Cosmic string	$\frac{\lambda}{4}\left(\lvert\varphi\rvert^2 - \eta^2\right)^2$	$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\varphi + \frac{\lambda}{2}\phi\left(\phi^2 - \eta^2\right) = 0$
Boson star	$\mu^2\lvert\varphi\rvert^2$	$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\varphi + (\mu^2 - \omega^2)\phi = 0$
Boson star	$\mu^2\lvert\varphi\rvert^2 + \frac{\lambda}{2}\lvert\varphi\rvert^4$	$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi + (\mu^2 - \omega^2)\phi - 2\lambda\phi^3 = 0$
Solitonic BS	$\mu^2\lvert\varphi\rvert^2\left(1 - \frac{2\lvert\varphi\rvert^2}{\sigma_0^2}\right)^2$	$\frac{\partial^2\phi}{\partial t^2} - \nabla^2\varphi + \mu^2\phi\left(1 - \frac{2\phi^2}{\sigma_0^2}\right)^2\left(1 - \frac{4\phi^2}{\sigma_0^2}\right) = 0$

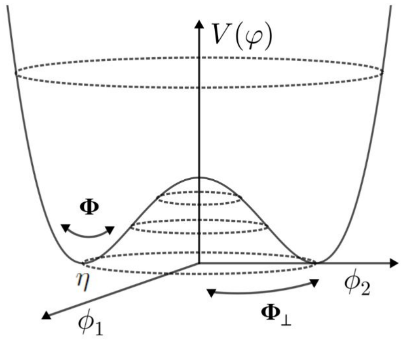
Analysis of real EL

Perturb about the vacuum state.

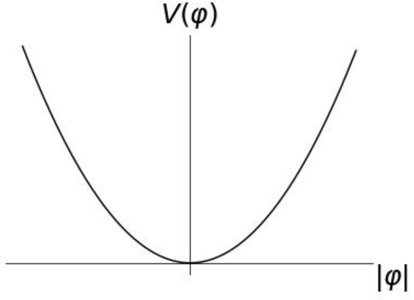
For cosmic strings, vary about minimum: $\phi = \eta + \chi$

For boson stars, vary about central minimum: $\phi = 0 + \chi$

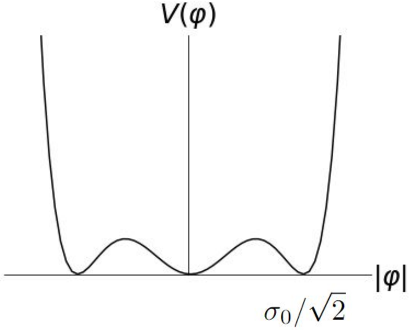
For solitonic boson stars, vary about outer minimum: $\phi = \sigma_0/\sqrt{2} + \chi$



Cosmic string



Boson star



Solitonic boson star

Analysis of real EL

Perturb real EL:

Type	Potential V	Perturbation
Cosmic string	$\frac{\lambda}{4}\left(\|\varphi\|^2 - \eta^2\right)^2$	$\frac{\partial^2\chi}{\partial t^2} - \nabla^2\chi + \lambda\eta^2\chi = 0$
Boson star	$\mu^2\|\varphi\|^2$	$\frac{\partial^2\chi}{\partial t^2} - \nabla^2\chi + (\mu^2 - \omega^2)\chi = 0$
Boson star	$\mu^2\|\varphi\|^2 + \frac{\lambda}{2}\|\varphi\|^4$	$\frac{\partial^2\chi}{\partial t^2} - \nabla^2\chi + (\mu^2 - \omega^2)\chi - 2\lambda\chi^3 = 0$
Solitonic BS	$\mu^2\|\varphi\|^2\left(1 - \frac{2\|\varphi\|^2}{\sigma_0^2}\right)^2$	$\frac{\partial^2\chi}{\partial t^2} - \nabla^2\chi - \frac{\mu^2\sqrt{2}}{\sigma_0}\chi^2 = 0$

Videos

Cosmic strings: https://youtube.com/playlist?list=PLSkfizpQDrcaTY_cPca9MtNTZ_K9mgsnP&feature=shared

Boson stars: https://www.youtube.com/watch?v=hj1WzNPpeS0

Conclusion

- Class of physical systems: complex scalar field with different potentials
- Similar methods used for analysing radiation
  - Separation into real and imaginary parts
  - Perturbation
- Different modes of radiation for cosmic strings and boson stars
  - Due to different symmetry in potential
- Future investigation:
  - Non-gravitational approach to initial profile of boson stars
  - Gravitational effects for boson stars
  - Gravitational waves

Citations

[1] Sidney R. Coleman. Q-balls. *Nucl. Phys. B*, 262(2):263, 1985. [Addendum: Nucl.Phys.B 269, 744 (1986)].

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[4] T. D. Lee and Y. Pang. Nontopological solitons. *Phys. Rept.*, 221:251–350, 1992.

[5] A. Vilenkin and E. P. S. Shellard. *Cosmic Strings and Other Topological Defects.* Cambridge University Press, 7 2000.