Communicating Mathematics: What are groups and why are they useful?

Justin Chen, as part of Plus Magazine (plus.maths.org) under Marianne Freiberger and Rachel Thomas

How to (im)prove mathematics

Find out how a story starting with the simple notion of counting ends in a revolutionary new way of doing maths that uses computers to harness the power of human collaboration!

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Maths in three minutes: Groups

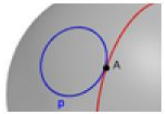
Capturing symmetry with algebra.

Maths in a minute: Symmetry

We all instinctively recognise symmetry w
think.

Emmy Noether and the power of symmetry

2018 is the centenary of a mathematical result that changec



Maths in a minute: The fundamental group

Topology considers two objects to be the same as long as
it. But how do you work with such a slippery concept? One
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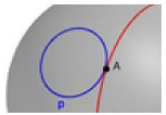

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I: What is a group?

After walking anywhere, you can continue walking in any other direction.

After walking anywhere, you can always walk back.

After rotating the ball in any way, you can continue rotating the ball in any other way.

After rotating the ball in any way, you can always rotate it back to where it started.

A *group* is a collection of actions which we can do one after the other, each of which can be undone.

A *symmetry* of an object is a transformation which leaves it looking the same as it started.

We can do symmetries one after the other, and any symmetry can always be undone.

We can do symmetries one after the other, and any symmetry can always be undone.

Therefore the set of symmetries of an object always forms a group.

Any group is also always the set of symmetries of some object.

Groups  Symmetries

II: How are groups used in mathematics?

A story from physics

A story from physics

Translational symmetry

A story from physics

A story from physics

Rotational symmetry

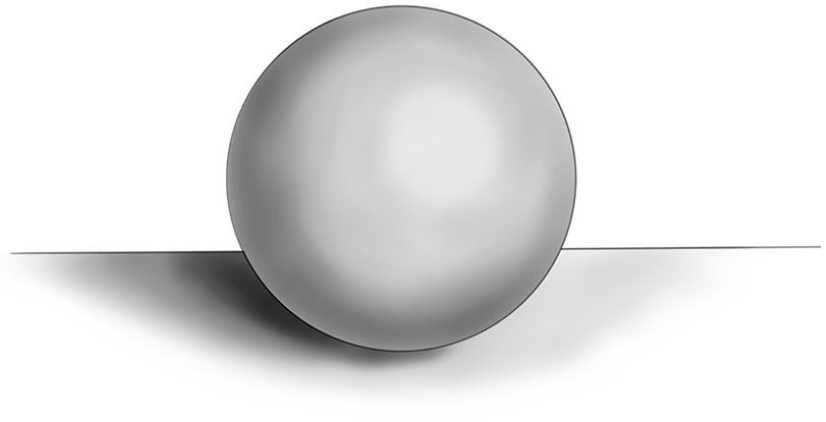
Symmetries of the universe correspond to laws of physics!

Translational symmetry → Conservation of momentum

Rotational symmetry → Conservation of angular momentum

A story from topology

A story from topology



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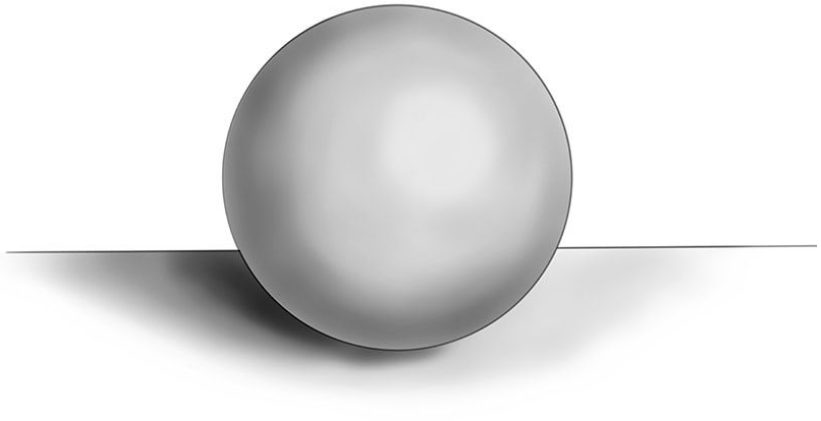
A story from topology



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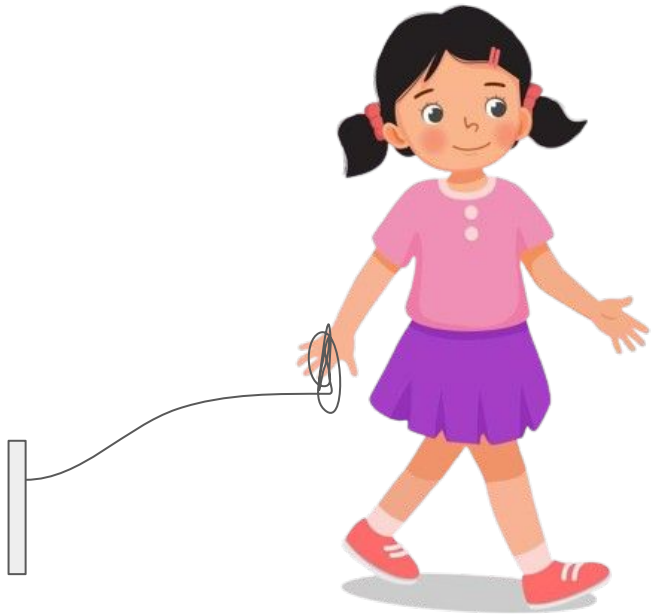
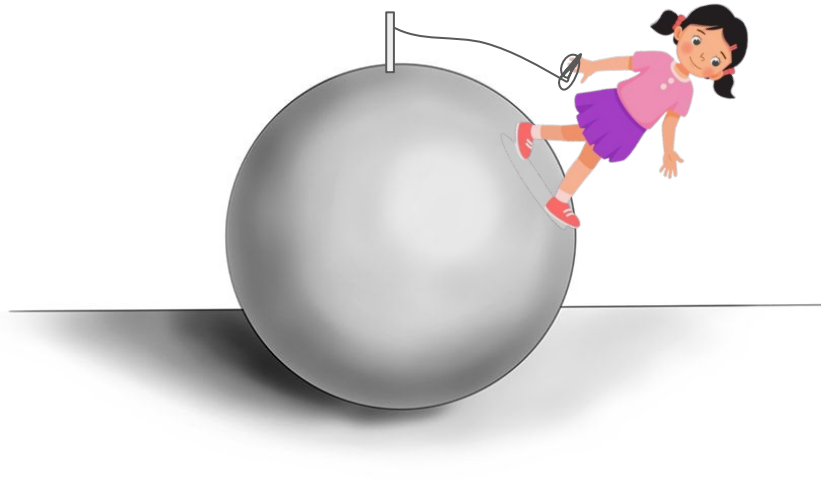
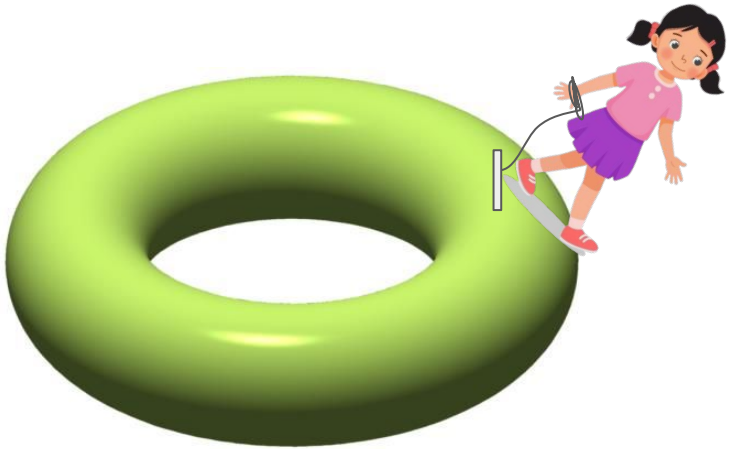


A story from topology





Groups are an important part of the language of modern mathematics and physics.

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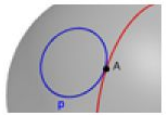

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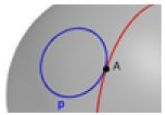

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How are they used?

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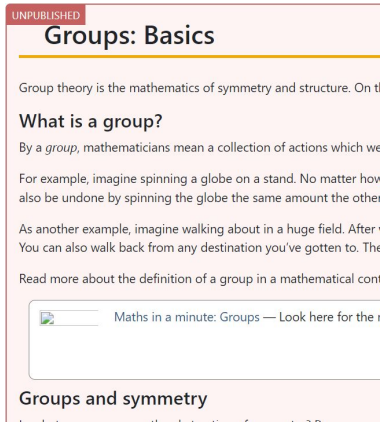
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Groups: Basics

Group theory is the mathematics of symmetry and structure. On t

What is a group?

By a *group*, mathematicians mean a collection of actions which we

For example, imagine spinning a globe on a stand. No matter hov
also be undone by spinning the globe the same amount the other

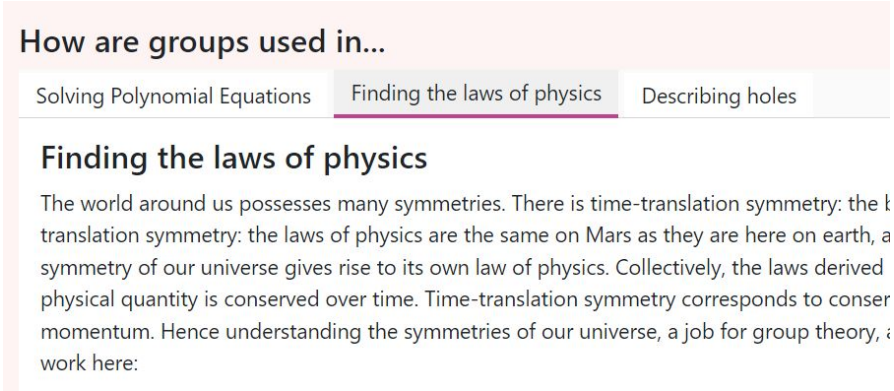
As another example, imagine walking about in a huge field. After
You can also walk back from any destination you've gotten to. The

Read more about the definition of a group in a mathematical cont

Maths in a minute: Groups — Look here for the

Groups and symmetry

How are they used?


How are groups used in...

Solving Polynomial Equations | Finding the laws of physics | Describing holes

Finding the laws of physics

The world around us possesses many symmetries. There is time-translation symmetry: the l
translation symmetry: the laws of physics are the same on Mars as they are here on earth, a
symmetry of our universe gives rise to its own law of physics. Collectively, the laws derived
physical quantity is conserved over time. Time-translation symmetry corresponds to conser
momentum. Hence understanding the symmetries of our universe, a job for group theory, a
work here:

Takeaways

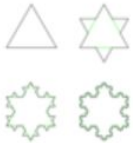
- Content can be organised at multiple levels
- On each level, tell a story:
  - Start from somewhere the audience can relate to
  - Keep a running metaphor/example
  - The same principles as with any maths talk, but with a different audience!

Other work

Articles

Maths in a minute: Measure

Learn how lengths, areas, and volumes generalise to the concept of *measure*, and how this relates to integration and probability.



How to compute the dimension of a fractal

Find out what it means for a shape to have fractional dimension.

Maths in a Minute: Group actions

What exactly do we mean when we say group theory is the study of symmetry? *Group actions* make precise what it means for a group to act by symmetries on an object.

Podcasts

Thank you!