

Communicating Mathematics: What are groups and why are they useful?

Justin Chen, as part of Plus Magazine (plus.maths.org) under
Marianne Freiberger and Rachel Thomas



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Find out how a story starting with the simple notion of counting ends in a revolutionary new way of doing maths that uses computers to harness the power of human collaboration!





Maths in three minutes: Groups

Capturing symmetry with algebra.



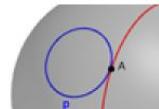
Maths in a minute: Symmetry

We all instinctively recognise symmetry without thinking.



Emmy Noether and the power of symmetry

2018 is the centenary of a mathematical result that changed the world.



Maths in a minute: The fundamental group

Topology considers two objects to be the same as long as they are homeomorphic to each other. But how do you work with such a slippery concept? One way is to look at a shape.



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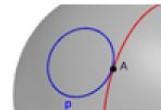
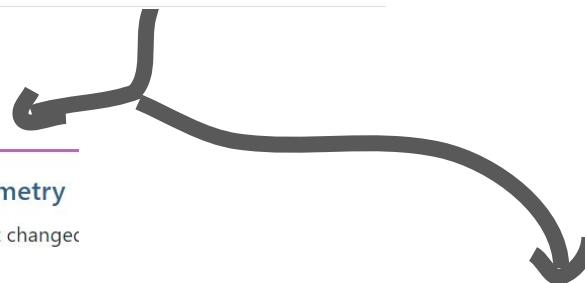
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I: What is a group?





After walking anywhere, you can continue walking in any other direction.

After walking anywhere, you can always walk back.





After rotating the ball in any way, you can continue rotating the ball in any other way.

After rotating the ball in any way, you can always rotate it back to where it started.

A group is a collection of actions which we can do one after the other, each of which can be undone.

A *symmetry* of an object is a transformation which leaves it looking the same as it started.



We can do symmetries one after the other, and any symmetry can always be undone.

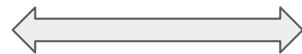
We can do symmetries one after the other, and any symmetry can always be undone.

Therefore the set of symmetries of an object always forms a group.

Any group is also always the set of symmetries of some object.



Groups



Symmetries

II: How are groups used in mathematics?

A story from physics



A story from physics



Translational symmetry

A story from physics



A story from physics



Rotational symmetry



Symmetries of the universe correspond to laws of physics!



Translational
symmetry



Conservation of
momentum



Rotational
symmetry

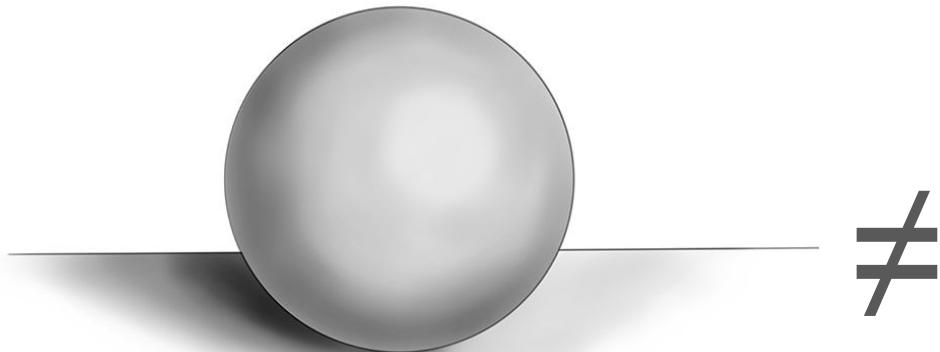


Conservation of
angular
momentum

A story from topology



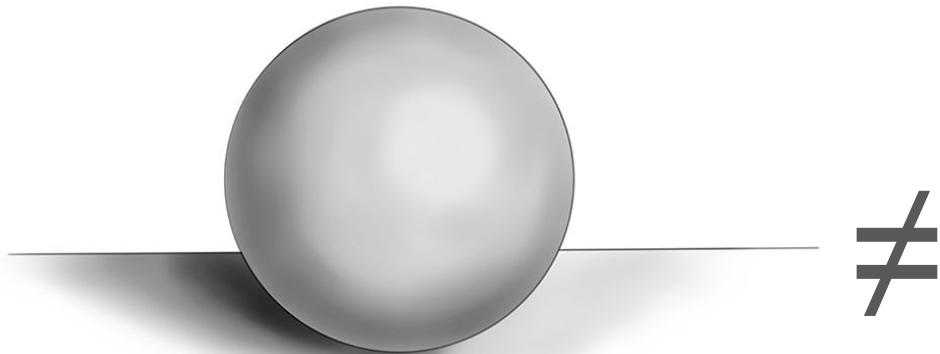
A story from topology



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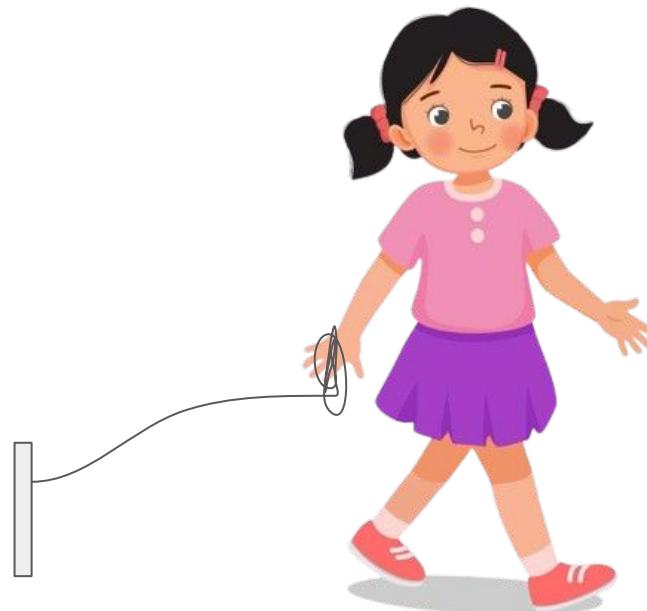
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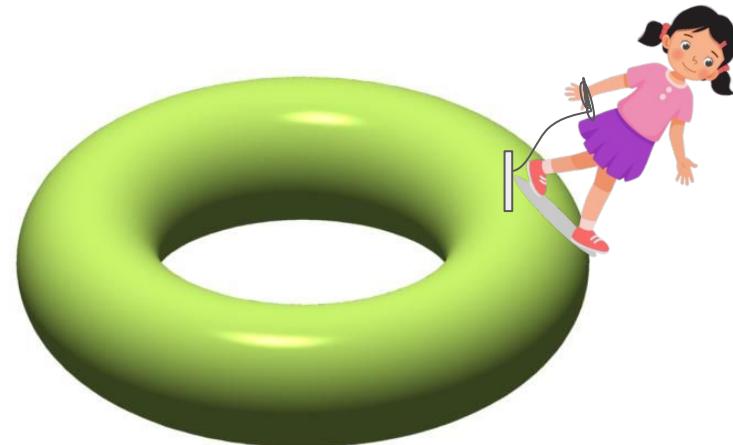
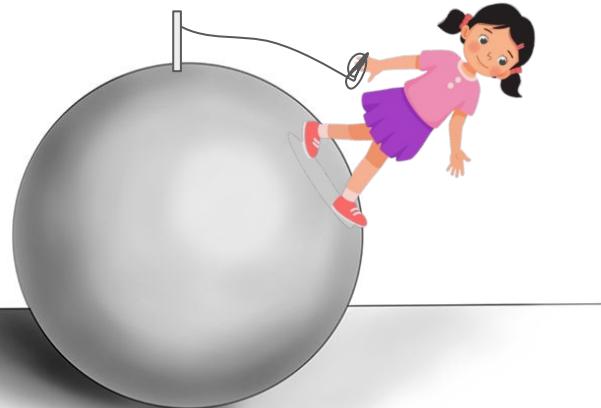
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A story from topology



A story from topology



Groups are an important part of the language of modern mathematics and physics.



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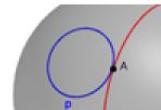
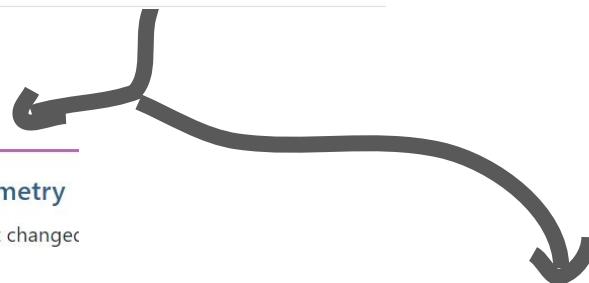
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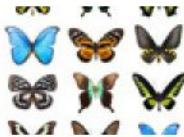
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Maths in a minute: The fundamental group

Topology considers two objects to be the same as long as they are homeomorphic to each other. But how do you work with such a slippery concept? One way is to consider loops.



Maths in three minutes: Groups

Capturing symmetry with algebra.

What is a group?



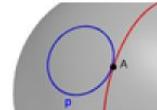
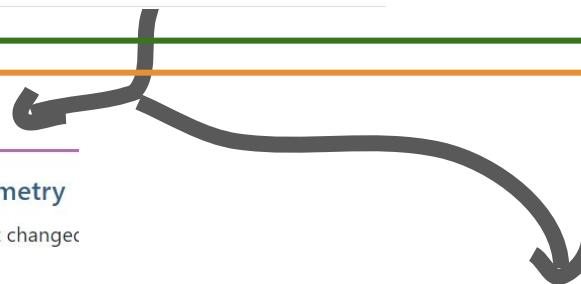
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How are they used?

UNPUBLISHED

Groups: Basics

Group theory is the mathematics of symmetry and structure. On this page, we'll explore what a group is, and how it's used in mathematics and physics.

What is a group?

By a *group*, mathematicians mean a collection of actions which we can combine in a sensible way.

For example, imagine spinning a globe on a stand. No matter how many times you spin it, it can always be undone by spinning the globe the same amount the other way.

As another example, imagine walking about in a huge field. After any walk, you can always walk back from any destination you've gotten to. The walk back is the inverse of the walk you took.

Read more about the definition of a group in a mathematical context.

 Maths in a minute: Groups — Look here for the full definition of a group.

Groups and symmetry

How are groups used in...

Solving Polynomial Equations

Finding the laws of physics

Describing holes

Finding the laws of physics

The world around us possesses many symmetries. There is time-translation symmetry: the laws of physics are the same at all times. There is space-translation symmetry: the laws of physics are the same on Mars as they are here on earth, and the same on the moon. The symmetry of our universe gives rise to its own law of physics. Collectively, the laws derived from this symmetry are called the laws of physics. A physical quantity is conserved over time. Time-translation symmetry corresponds to conservation of energy. Space-translation symmetry corresponds to conservation of momentum. Hence understanding the symmetries of our universe, a job for group theory, is the key to understanding the laws of physics.

How are they used?

What is a group?

Takeaways

- Content can be organised at multiple levels
- On each level, tell a story:
 - Start from somewhere the audience can relate to
 - Keep a running metaphor/example
 - The same principles as with any maths talk, but with a different audience!

Other work

Articles



Maths in a minute: Measure

Learn how lengths, areas, and volumes generalise to the concept of *measure*, and how this relates to integration and probability.



How to compute the dimension of a fractal

Find out what it means for a shape to have fractional dimension.



Maths in a Minute: Group actions

What exactly do we mean when we say group theory is the study of symmetry? *Group actions* make precise what it means for a group to act by symmetries on an object.

Podcasts

The screenshot shows a podcast player interface. At the top, there is a small image of a dog wearing sunglasses and a straw hat. To the right of the image, the text 'Maths on the Move' is displayed. Below this, the title of the episode is 'The secret to a happy summer: Seville, causality and staircases!'. Underneath the title, there is a green play button icon and a progress bar showing '0:00'. To the right of the progress bar, there are several icons: a Wi-Fi signal, a download arrow, a share icon, and a volume icon. At the bottom of the player, there is a purple bar with a white waveform representing the audio content.

Thank you!