Get Out from Underneath

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The set-up

-Get out from Underneath is a pursuit evasion game played on a n by n grid. A team of cops vs one fast robber.

-The cops pick the starting position.

-The team of cops need to stay on top of a robber.

-How many cops do we need to ensure this?

1D game

-Exact answer in $\left\lceil\frac{n}{2}\right\rceil$.

-Cops’ strategy is for each of them to guard two adjacent cells.

-Robber’s strategy is to sprint across.

Back to 2D

-These strategies translate to a lower bound of n and an upper bound of $\frac{n^2}{4}$.

-In my project I improved these to $n^{1.35}$ and $n^{1.999}$ respectively (exponents not exact)

The new cop strategy

-What seemed hard?

-o(n^2), density must go to 0.

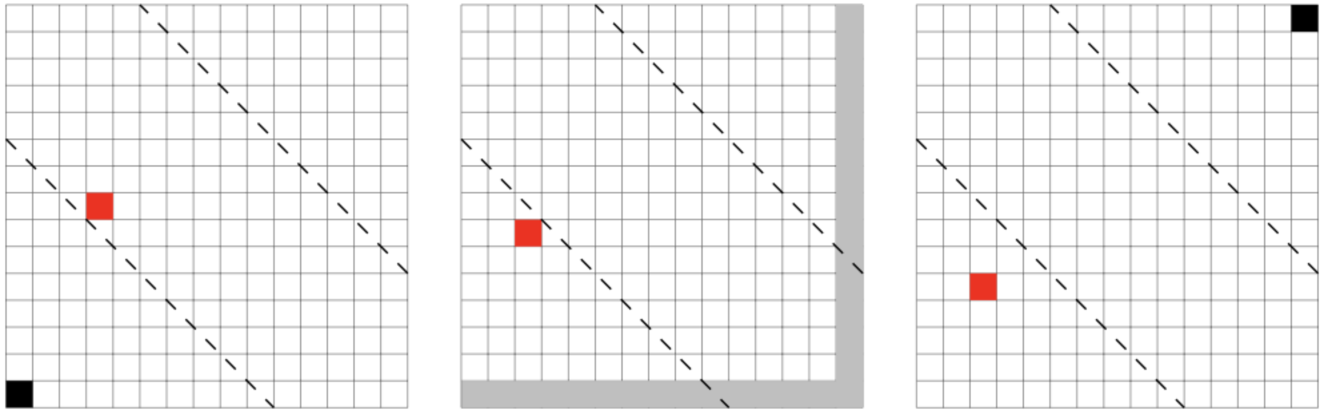
-Can play on a torus, tells us we need asymmetry.

The new cop strategy

-We can catch the robber with $k^2 - 1$ cops where the cops only move when the robber is far from them.

-We do this by using that 'no-cop' can move faster than a cop and exploiting the 2 dimensions.

(Black: no cop, white: stationary cop, grey: moving cop)

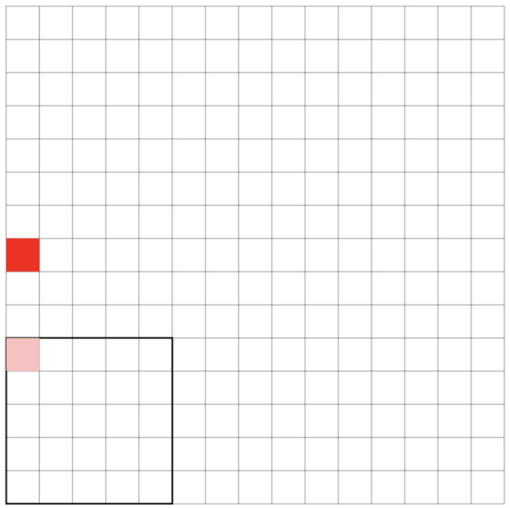


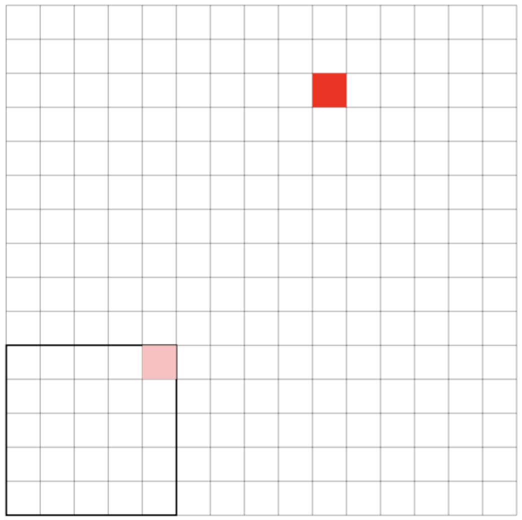
The new cop strategy

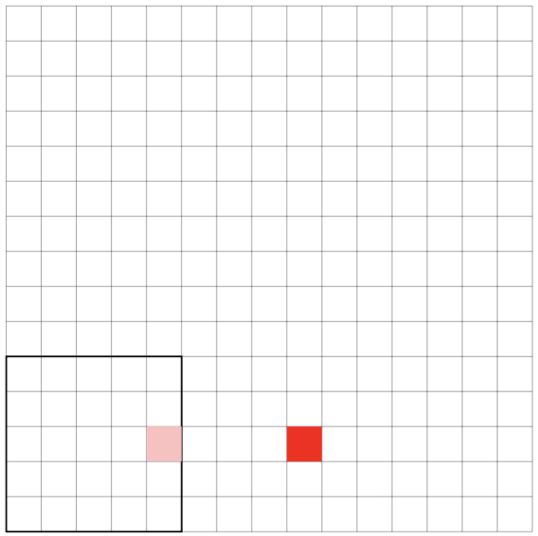
-The second idea is that we can build cop strategies from smaller cop strategies.

-A team of cops play on a sub-square to catch a phantom robber.

-Tiling the plane with these sub squares, we build a strategy for the bigger square







The new cop strategy

-Tile 15n by 15n grid with 224 teams, leaving one empty.

-These teams move as in the 15 by 15.

-How does moving a team of cops work?

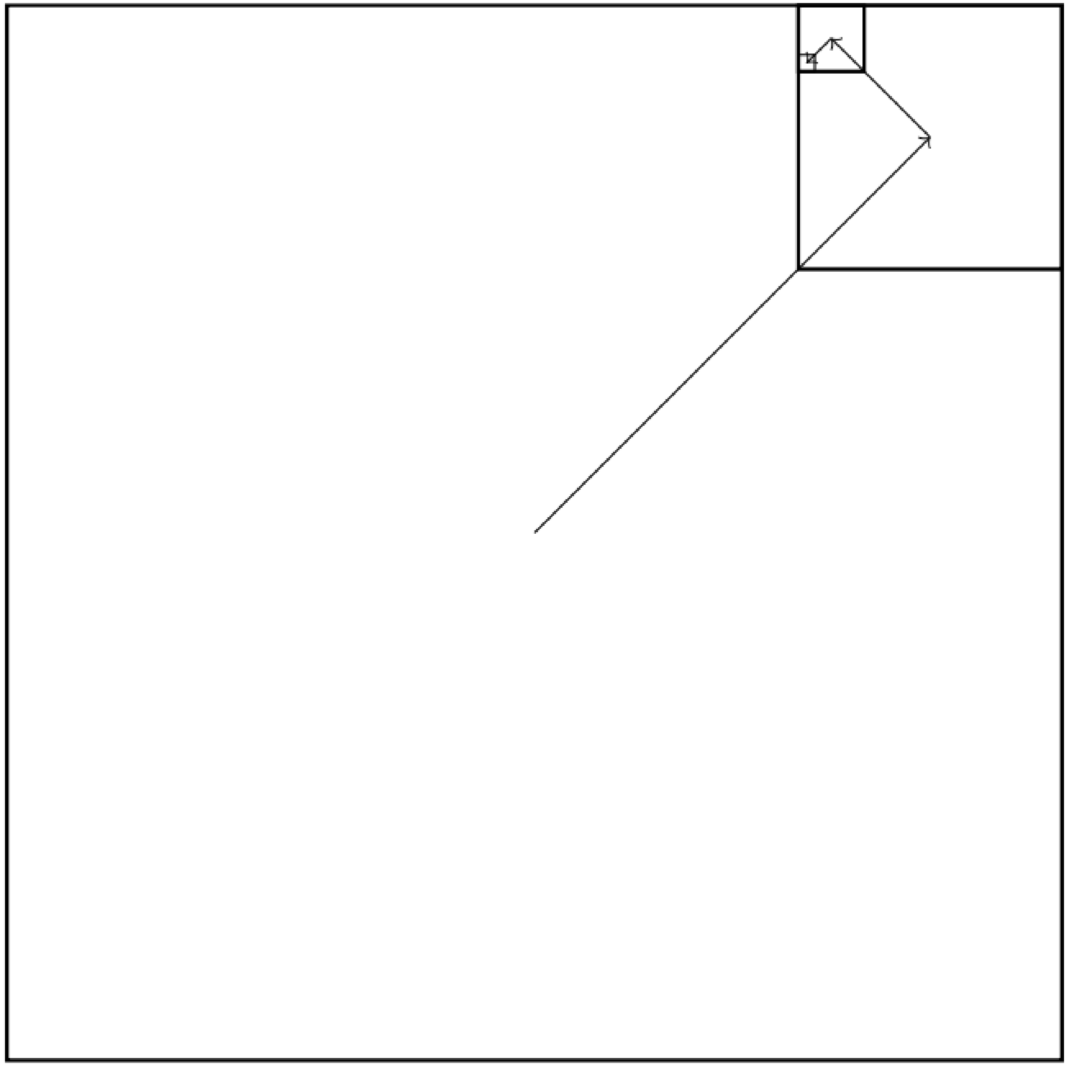
-It gives us an exponent of $\log_{15} 224$, approx. 1.998.

Higher dimensions

-What about higher dimensions?

-Lower bound of $n^{d/2}$.

-Iteratively run into quadrants with fewest cops.



Higher dimensions

-What about an upper bound?

-Similar strategy to before shows it is $o(n^d)$.

-We can do a lot better.

-We first need to answer the ‘fixed-time’ question

Higher dimensions

-The cops split up into 5n/4 teams of $\sim n^{d/2}$.

-Each team catches the robber at times i modulo 5n/4.

-Gives an upper bound of $\sim n^{\frac{d}{2}+1}$.

Higher dimensions

Question: Is either $n^{d/2}$ or $n^{\frac{d}{2}+1}$ the 'right answer' as d goes to infinity?

Questions?