Hydrothermal circulation near mid-ocean ridges

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The geophysical problem

A key phenomenon of the Earth's oceans is that seawater is constantly driven in and out of the porous rock under the ocean floor. This 'hydrothermal circulation' impacts the transfer of heat and chemical composition of our world. The activity is concentrated around mid-ocean ridges—regions where tectonic plates are moving apart, allowing magma to rise and provide stronger heating. This increased intensity induces convection, which is the transfer of heat via the circulation of water. The nature of the fluid velocity and temperature distribution within these particularly active regions of porous oceanic crust is of interest to geophysicists, but difficult to physically investigate and is immediately thermodynamically complex. Thus, despite the vastness of the phenomenon, the generalised behaviour of convection near mid-ocean ridges is not thoroughly understood. By approaching the problem from a fluid dynamics perspective, we aimed to develop and interpret a mathematical model to understand its qualitative features.

The mathematical model

The rock through which fluid can circulate is modelled as an infinitely wide two-dimensional layer below the ocean floor of dimensionless height $H = 1$. This strip is heated below by magma, inducing a temperature gradient. We then impose an impermeability condition on the bottom boundary and assume that the ocean exerts a constant pressure on the rock at the top. (Note, since water cannot flow continuously throughout the rock but instead must follow its pores, our 'velocity' is more accurately described as the average volume flux over some small region.) Finally, the governing dimensionless equations describing the conservation of mass, momentum and heat in our system are given by

$$\nabla \cdot \mathbf{u} = 0, \quad \mathbf{u} = -\nabla p + T\mathbf{e_z}, \quad \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{\mathrm{Ra}} \nabla^2 T,$$

where $\mathbf{u} = (u, 0, w)$ is 'velocity', T is temperature, p is pressure and Ra is the Rayleigh number. The Rayleigh number is a dimensionless coefficient representing the strength of buoyancy-driven forces against dissipative effects. It is notably proportional to the temperature difference across the layer of porous rock, and hence can be interpreted as the effective strength of heating from the magma with respect to all other fluid and rock properties of a particular system. The Rayleigh number characterises the nature of convection. For example, when the base magmatic heating is uniform, i.e. $T(x, z = 0) = 1$ and $T(x, z = 1) = 0$, we can analytically determine that convection only occurs when the Rayleigh number is above the critical value $\mathrm{Ra}_{\mathrm{crit}} = 27.10$. Convection under these boundary conditions looks like steady, discrete 'cells' of circulating fluid. To investigate our model at various Rayleigh numbers, we turned to numerical simulations (the version discussed in this summary solved the coupled governing equations using finite difference methods and time-stepping schemes). Transitioning to numerical schemes provided the flexibility to explore interesting iterations on the original model, informed by seismic and drill site data taken from a mid-ocean ridge.

Investigating a varied base temperature distribution

One direction involved observing how the base magmatic heating diminished with distance from the ridge—hence treating the ridge as a localised heat source and modelling the bottom temperature boundary condition as a Gaussian curve with some standard deviation σ. Enforcing this central temperature plume immediately changes the character of how heat is transferred. Firstly, convection—and thus the circulation of water—will always occur, even at low Rayleigh numbers. Additionally, as the Rayleigh number increases and further temperature plumes attempt to form, the system falls into periodic quasi-steady states, as plumes are washed into the central plume. The nature of the periodicity can be tracked by considering a Nusselt number against time plot (see Figure 1). The Nusselt number is the measure of dimensionless flux out of the top of our 'box', quantifying how much water shoots from the rock into the ocean due to convection. We can determine the qualitative regimes the Nusselt number exhibits as the convection becomes more vigorous and the plume structure breaks down. As expected, the behaviour depends on the standard deviation of the Gaussian boundary condition.

Investigating permeability variation

An additional model we explored was introducing rock permeability, a property which measures how easily fluid can flow through the porous medium. Rock permeability, and therefore freedom of fluid movement, decreases as we go down into the crust. This vertical variation was implemented by considering an exponentially decreasing permeability function. Under these conditions, we observed the convection cells shift upwards, since fluid had more difficulty circulating in the bottom regions. In addition, the critical Rayleigh number increased with the strength of the decay, as stronger buoyancy forces are needed to induce convection.

Outlook

Our project produced models and metrics to track the qualitative behaviour of convection. It would be pertinent to continue to refine and automate our data collection and analysis. As with any attempt to model the real world, there is always room for additional physics and the relaxation of assumptions, which we can hope to consider once our current models are thoroughly understood. For example, an immediate next step would be combining the localised heat source boundary condition with a varying rock permeability.

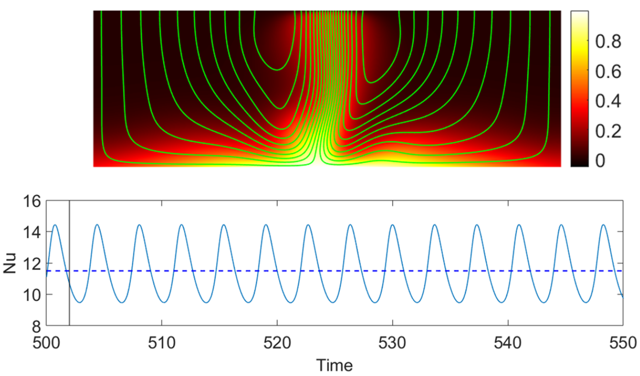

Figure 1: *Top*: A snapshot of a simulation with aspect ratio 3 and Gaussian base heating with $\sigma = 1$, at $\mathrm{Ra} = 250$. The 'velocity' field streamlines are green lines, and the temperature field is given by the colour map. At this Ra, the central plume is prominent, but not symmetrical. *Bottom*: The corresponding Nusselt number versus time graph during the final state of the simulation. It exhibits clear periodicity.