Summary of Summer Project: Non-Circular Discs in Binary Systems

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This project explores the dynamics of non-circular gaseous discs in binary systems by studying periodic orbits of test particles in the rotating frame. The goal is to connect numerical simulations with analytical theories, and ultimately understand non-axisymmetric features in accretion or debris discs around binary stars.

1. Numerical Construction of Periodic Orbits
We numerically integrate the planar equations of motion using the classical fourth-order Runge–Kutta (RK4) method. Initial conditions are set such that the particle starts on the x-axis with zero initial x-velocity ($\dot{x} = 0$), and an initial guess for y-velocity ($\dot{y} = v_y$). To locate periodic orbits (POs), we apply the secant method, using the Keplerian circular velocity as an initial guess for v_y. This yields a family of closed orbits parameterized by the initial x-position x_0, from which the corresponding v_y is determined numerically.

2. Fourier Decomposition and Comparison with Theory
Once the periodic orbits are constructed, we transform each trajectory into polar coordinates and extract the radius $r(\phi)$ as a function of phase angle. We compute the Fourier coefficients of $r(\phi)$ numerically and compare them with theoretical predictions from two analytical frameworks: linear perturbation theory and secular theory. While general agreement is observed for several harmonics, the r_{11} mode shows significant discrepancies, suggesting the need for a refined theoretical model.

3. Complex Representation and Hill's Approximation
To better understand the structure of periodic orbits, we consider their complex representation $\zeta = x + iy$ and express the motion in Fourier space as

$$\zeta(\phi) = \sum_{m=-2}^{2} \zeta_m \, e^{im\phi},$$

where ϕ is the orbital phase. In the limit $\mu \to 1$, corresponding to Hill's problem, a theoretical bifurcation is predicted at $\Omega \approx 5.97$. We investigate the behavior of $|\zeta_m|$ as a function of Ω and compare the numerical results against asymptotic predictions derived under Hill's approximation. The log–log plots reveal power-law scalings for various m-modes, and help identify the limits of the linear theory near bifurcation.

This ongoing project aims to bridge high-precision numerical integration with analytical insight, potentially contributing to the understanding of asymmetric features in astrophysical discs.

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