Focusing Phenomena for Wave Propagation in Random Media

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Multiply scattering random media give rise to striking focusing phenomena such as narrow, high-intensity ribbons in the propagating wavefield. These features are interesting to predict because the wave dynamics involve both diffraction and random phase modulation.

This summer I worked on a numerical study of wavefront propagation in such media using the parabolic split-step wave equation. Our aim was to reproduce the ribbon-like intensity patterns reported in the literature and to investigate simplified models that could capture the key transport behaviour without the heavy computational cost of full simulations.

The first stage of the project was to construct random phase screens that represent the refractive-index fluctuations of a weakly scattering medium. Next we implemented a split-step Fourier method for solving the Wave Equation. Using these screens, we then propagated a Gaussian beam across a two-dimensional grid with the solver. Each marching step applied the random phase modulation and then handled diffraction in Fourier space. This algorithm allowed us to produce detailed maps of the evolving intensity pattern. From these simulations we observed clear transverse spreading of beams and identified regions of strong local focusing that formed the high-intensity ribbons of interest.

To avoid repeatedly running the full solver we developed reduced transport models. In a “ball-bin shooting” approach we used the directional drifts of intensity measured in the simulations to define transition probabilities for a Markov process that moves particles across the medium. By averaging over many random input wavefronts and source positions we built up a presence map that closely matched the high-intensity structures seen in the full wave simulations.

We also tested a purely curvature-based method, in which the local phase curvature of the medium predicted the change in variance of lateral displacement after scattering. We found a strong correlation between local curvature and beam spreading, and this approach reproduced the key focusing patterns with far less computation than the full solver.

We are yet to work on fine-tuning our model and looking into higher dimensional properties of the intensity, e.g. the fourth moment of the field, used to study scintillation.

I also explored a possible biomedical application: modelling transcranial ultrasonic stimulation (TUS) using the MATLAB toolbox k-Wave. I studied simple simulations of ultrasound propagation through a CT-based skull mask to examine pressure fields and estimate cavitation thresholds. This gave me a first look at how the numerical techniques developed in this project can be adapted to medical-ultrasound problems.

This project gave me hands-on experience in fast random-field generation, FFT-based numerical propagation, and in comparing detailed physical simulations with stochastic reduced models. Our results show that relatively simple drift- or curvature-based descriptions can enhance the essential scattering patterns from the research paper we referenced.