Report of the Curriculum Committee: 1

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Irena Borzym, Shilin Lai, Henrik Latter, John Lister, Clément Mouhot, Thomas Zielinski, Michael Tehranchi (chair)

Various changes to *The Schedules of the Mathematical Tripos* that have been considered by the Curriculum Committee during the 2015-16 year. This document lists those changes to existing courses that the Committee has agreed at its meetings on 29 October 2015 and 4 February 2016. A preliminary version of this document was circulated at the Easter term 2016 staff meetings of both DPMMS and DAMTP. Only those proposals which seemed well-received by both departments are listed here. A second document containing the Committee's views on the remaining, more controversial, proposals is forthcoming.

1 IA Vector Calculus

Proposed change: 'Radius of curvature' to become 'curvature and torsion' in the first section.

The change was proposed in *Final Report of the Examiners Mathematical Tripos Part IA, 2015* on the grounds that this topic is already lectured and examined. There was a further proposal from the Committee to add the 'Frenet-Serret formulae' to the first paragraph, since they are implicit in the definition of torsion. However, the majority view was not to insist that they appear explicitly in the schedule: although a lecturer is free to mention them, their explicit inclusion might distract from the main aim of the first lecture and would add another examinable fact for students to memorise.

2 IA Vectors & Matrices

Proposed additions: 'rank-nullity theorem (statement only)' to the third section.

This change was also proposed in *Final Report of the Examiners Mathematical Tripos Part IA, 2015* on the grounds that this topics is already lectured and examined. The Committee agrees that the statement of rank-nullity is sufficiently easy and useful to be included in the schedule. However, the Committee recommends that the proof of this result remain in IB Linear Algebra.

The IA Examiners also proposed the inclusion of the Gram–Schmidt procedure. The Committee does not recommend this change: Although general n-dimensional vectors are only to be discussed briefly, the main focus of the course is on the concrete cases of dimensions two and three. The Gram–Schmidt procedure is already included in IB Linear Algebra, where the Committee recommends it remain.

3 IB Linear Algebra

Proposed addition: 'quotient spaces' to the first paragraph

This proposal is from Dr Wadsley, the current lecturer. Since quotient groups are covered in IA, we agree that adding 'quotient spaces' to the first paragraph is a sensible suggestion. It is an important concept in Linear Algebra, not requiring a great deal of extra time to lecture, and would tie in well with quotient modules in the Group, Rings & Modules course.

4 IIC Automata & Formal Languages

Proposed deletions: 'Post-systems', 'λ-calculus' from the first section.'Myhill–Nerode theorem', 'two-way finite automata' from the second section. 'Greibach normal form', 'left- and right-grammars' from the third section. All of the fourth section.

Proposed additions: 'Universal register machines', 'the recursion theorem', the 's-m-n theorem' to the first section.

These changes were proposed by Dr Chiodo, the current lecturer. The deletions are to prevent this course from being over-stuffed in light of its IIC status. The additions are not hard to prove, but are fundamental to computability theory. The precise wording has been checked by Dr Chiodo.

5 IID General Relativity

Proposed change: to star the proof of Birkhoff's theorem.

This proposal is from Dr Siklos, who notes that the proof is rather long and very difficult to examine.

VECTOR CALCULUS **24 lectures, Lent term**

Curves in $\mathbb{R}^3$
Parameterised curves and arc length, tangents and normals to curves in $\mathbb{R}^3$; curvature and torsion. [1]

Integration in $\mathbb{R}^2$ and $\mathbb{R}^3$
Line integrals. Surface and volume integrals: definitions, examples using Cartesian, cylindrical and spherical coordinates; change of variables. [4]

Vector operators
Directional derivatives. The gradient of a real-valued function: definition; interpretation as normal to level surfaces; examples including the use of cylindrical, spherical *and general orthogonal curvilinear* coordinates.

Divergence, curl and ∇^2 in Cartesian coordinates, examples; formulae for these operators (statement only) in cylindrical, spherical *and general orthogonal curvilinear* coordinates. Solenoidal fields, irrotational fields and conservative fields; scalar potentials. Vector derivative identities. [5]

Integration theorems
Divergence theorem, Green's theorem, Stokes's theorem, Green's second theorem: statements; informal proofs; examples; application to fluid dynamics, and to electromagnetism including statement of Maxwell's equations. [5]

Laplace's equation
Laplace's equation in $\mathbb{R}^2$ and $\mathbb{R}^3$: uniqueness theorem and maximum principle. Solution of Poisson's equation by Gauss's method (for spherical and cylindrical symmetry) and as an integral. [4]

Cartesian tensors in $\mathbb{R}^3$
Tensor transformation laws, addition, multiplication, contraction, with emphasis on tensors of second rank. Isotropic second and third rank tensors. Symmetric and antisymmetric tensors. Revision of principal axes and diagonalization. Quotient theorem. Examples including inertia and conductivity. [5]

Appropriate books

H. Anton *Calculus.* Wiley Student Edition 2000
T.M. Apostol *Calculus.* Wiley Student Edition 1975
M.L. Boas *Mathematical Methods in the Physical Sciences.* Wiley 1983
† D.E. Bourne and P.C. Kendall *Vector Analysis and Cartesian Tensors.* 3rd edition, Nelson Thornes 1999
E. Kreyszig *Advanced Engineering Mathematics.* Wiley International Edition 1999
J.E. Marsden and A.J.Tromba *Vector Calculus.* Freeman 1996
P.C. Matthews *Vector Calculus.* SUMS (Springer Undergraduate Mathematics Series) 1998
† K. F. Riley, M.P. Hobson, and S.J. Bence *Mathematical Methods for Physics and Engineering.* Cambridge University Press 2002
H.M. Schey *Div, grad, curl and all that: an informal text on vector calculus.* Norton 1996
M.R. Spiegel *Schaums outline of Vector Analysis.* McGraw Hill 1974

VECTORS AND MATRICES **24 lectures, Michaelmas term**

Complex numbers

Review of complex numbers, including complex conjugate, inverse, modulus, argument and Argand diagram. Informal treatment of complex logarithm, n-th roots and complex powers. de Moivre's theorem. [2]

Vectors

Review of elementary algebra of vectors in $\mathbb{R}^3$, including scalar product. Brief discussion of vectors in $\mathbb{R}^n$ and $\mathbb{C}^n$; scalar product and the Cauchy-Schwarz inequality. Concepts of linear span, linear independence, subspaces, basis and dimension.

Suffix notation: including summation convention, δ_{ij} and ϵ_{ijk}. Vector product and triple product: definition and geometrical interpretation. Solution of linear vector equations. Applications of vectors to geometry, including equations of lines, planes and spheres. [5]

Matrices

Elementary algebra of 3×3 matrices, including determinants. Extension to $n \times n$ complex matrices. Trace, determinant, non-singular matrices and inverses. Matrices as linear transformations; examples of geometrical actions including rotations, reflections, dilations, shears; kernel and image, rank-nullity theorem (statement only). [4]

Simultaneous linear equations: matrix formulation; existence and uniqueness of solutions, geometric interpretation; Gaussian elimination. [3]

Symmetric, anti-symmetric, orthogonal, hermitian and unitary matrices. Decomposition of a general matrix into isotropic, symmetric trace-free and antisymmetric parts. [1]

Eigenvalues and Eigenvectors

Eigenvalues and eigenvectors; geometric significance. [2]

Proof that eigenvalues of hermitian matrix are real, and that distinct eigenvalues give an orthogonal basis of eigenvectors. The effect of a general change of basis (similarity transformations). Diagonalization of general matrices: sufficient conditions; examples of matrices that cannot be diagonalized. Canonical forms for 2×2 matrices. [5]

Discussion of quadratic forms, including change of basis. Classification of conics, cartesian and polar forms. [1]

Rotation matrices and Lorentz transformations as transformation groups. [1]

Appropriate books

Alan F Beardon *Algebra and Geometry.* CUP 2005

† Gilbert Strang *Linear Algebra and Its Applications.* Thomson Brooks/Cole 2006

Richard Kaye and Robert Wilson *Linear Algebra.* Oxford Science Publications 1998

D.E. Bourne and P.C. Kendall *Vector Analysis and Cartesian Tensors.* Nelson Thornes 1992

E. Sernesi *Linear Algebra: A Geometric Approach.* CRC Press 1993

James J. Callahan *The Geometry of Spacetime: An Introduction to Special and General Relativity.* Springer 2000

LINEAR ALGEBRA **24 lectures , Michaelmas term**

Definition of a vector space (over $\mathbb{R}$ or $\mathbb{C}$), subspaces, the space spanned by a subset. Linear independence, bases, dimension. Direct sums and complementary subspaces. Quotient spaces. [3]

Linear maps, isomorphisms. Relation between rank and nullity. The space of linear maps from U to V, representation by matrices. Change of basis. Row rank and column rank. [4]

Determinant and trace of a square matrix. Determinant of a product of two matrices and of the inverse matrix. Determinant of an endomorphism. The adjugate matrix. [3]

Eigenvalues and eigenvectors. Diagonal and triangular forms. Characteristic and minimal polynomials. Cayley–Hamilton Theorem over $\mathbb{C}$. Algebraic and geometric multiplicity of eigenvalues. Statement and illustration of Jordan normal form. [4]

Dual of a finite-dimensional vector space, dual bases and maps. Matrix representation, rank and determinant of dual map [2]

Bilinear forms. Matrix representation, change of basis. Symmetric forms and their link with quadratic forms. Diagonalisation of quadratic forms. Law of inertia, classification by rank and signature. Complex Hermitian forms. [4]

Inner product spaces, orthonormal sets, orthogonal projection, $V = W \oplus W^{\perp}$. Gram–Schmidt orthogonalisation. Adjoints. Diagonalisation of Hermitian matrices. Orthogonality of eigenvectors and properties of eigenvalues. [4]

Appropriate books

C.W. Curtis *Linear Algebra: an introductory approach.* Springer 1984
P.R. Halmos *Finite-dimensional vector spaces.* Springer 1974
K. Hoffman and R. Kunze *Linear Algebra.* Prentice-Hall 1971

AUTOMATA AND FORMAL LANGUAGES (C) **24 lectures, Michaelmas term**

Part IA Numbers and Sets is essential.

Register machines
Definition of a register machine; recursive functions, recursively enumerable sets. Church's thesis. Undecidability of the halting problem for register machines. Universal register machines. The recursion theorem. The s-m-n theorem. Other undecidable problems: reduction and Rice's theorem. [9]

Regular languages and finite-state automata
Deterministic finite-state automata (DFA) and regular languages. Nondeterminism. Regular expressions. Limitations of finite-state automata: homomorphisms and closure properties; the pumping lemma; examples of non-regular languages. Minimization and the use of equivalence relations and quotients to construct minimal DFAs. [9]

Pushdown automata and context-free languages
Context-free grammars and context-free languages: generating languages by replacement rules. Normal forms for context-free languages, the Chomsky normal form; regular languages are context-free. Limitations of context-free grammars: the pumping lemma for context-free languages; examples of non-context-free languages. Pushdown automata. [6]

Appropriate books

J.E. Hopcroft, R. Motwani and J.D. Ullman *Introduction to automata theory, languages and computation, 3rd edn.* Pearson 2006 (£82.20 paperback)

P.T. Johnstone *Notes on logic and set theory (Chapter 4).* CUP 1987 (£15.95 paperback)

† D.C. Kozen *Automata and computability.* Springer 1997 (£48.00 paperback).

R.I. Soare *Recursively enumerable sets and degrees: a study of computable functions and computably generated sets.* Springer (Perspectives in mathematical logic) 1987 (£58.33 paperback)

M. Sipser *Introduction to the theory of computation.* Wadsworth Publishing Co 2012 (£141.00 hardback; £57 paperback)

GENERAL RELATIVITY (D) **24 lectures, Lent term**

Part IB Methods and Variational Principles are very useful.

Brief review of Special Relativity
Notion of proper time. Equation of motion for free point particle derivable from a variational principle. Noether's Theorem. [1]

Introduction and motivation for General Relativity
Curved and Riemannian spaces. The Pound-Rebka experiment. Introduction to general relativity: interpretation of the metric, clock hypothesis, geodesics, equivalence principles. Static spacetimes. Newtonian limit. [4]

Tensor calculus
Covariant and contravariant tensors, tensor manipulation, partial derivatives of tensors. Metric tensor, magnitudes, angles, duration of curve, geodesics. Connection, Christoffel symbols, covariant derivatives, parallel transport, autoparallels as geodesics. Curvature. Riemann and Ricci tensors, geodesic deviation. [5]

Vacuum field equations
Spherically symmetric spacetimes, the Schwarzschild solution. Birkhoff's Theorem *with proof*. Rays and orbits, gravitational red-shift, light deflection, perihelion advance. Shapiro time delay. [4]

Einstein Equations coupled to matter
Concept of an energy momentum tensor. Maxwell stress tensor and perfect fluid as examples. Importance of Bianchi identities. The emergence of the cosmological term. Simple exact solutions: Friedmann–Lemaitre metrics, the Einstein Static Universe. Hubble expansion and redshift. De-Sitter spacetime, mention of Dark Energy and the problem of Dark matter. Notion of geodesic completeness and definition of a spacetime singularity. Schwarzschild and Friedmann–Lemaitre spacetimes as examples of spacetimes with singularities. [4]

Linearized theory
Linearized form of the vacuum equations. De-Donder gauge and reduction to wave equation. Comparison of linearized point mass solution with exact Schwarzschild solution and identification of the mass parameter. Gravitational waves in linearized theory. *The quadrupole formula for energy radiated.* Comparison of linearized gravitational waves with the exact pp-wave metric. [4]

Gravitational collapse and black holes
Non-singular nature of the surface $r = 2M$ in the Schwarzschild solution using Finkelstein and Kruskal coordinates. The idea of an event horizon and the one-way passage of timelike geodesics through it. Qualitative account of idealized spherically symmetric collapse. The final state: statement of Israel's Theorem. *Qualitative description of Hawking radiation.* [2]

Appropriate books

S.M. Carroll *Spacetime and Geometry.* Addison-Wesley 2004
J.B. Hartle *Gravity: An introduction to Einstein's General Relativity.* AddisonWesley 2002
L.P. Hughston and K.P. Tod *An Introduction to General Relativity.* Cambridge University Press 1990
R. d'Inverno *Introducing Einstein's Relativity.* Clarendon 1992
† W. Rindler *Relativity: Special, General and Cosmological.* Oxford University Press 2001
H. Stephani *Relativity: An introduction to Special and General Relativity.* Cambridge University Press, 2004