Asymptopia

CENTRE FOR MATHEMATICAL SCIENCES NEWSLETTER

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The Betty and Gordon Moore Library

Michael Wilson, Cambridge University Library

The circular concrete shell of the Betty and Gordon Moore Library has risen spectacularly during the summer months. After careful planning and consultation the Moore Library will be operational next October. When the doors open for the first time what can the visitor expect to see?

The Moore Library has been designed as a working science library to meet the needs of both present and future generations of mathematicians and physical scientists. It will stand as a benchmark 'hybrid' library, combining access to both print and electronic information.

There are several key requirements to delivering quality services within this hybrid model which have been consciously built in to the design.

First is the need to accommodate growing conventional print collections which, despite forecasts of an early demise at the hands of the internet, shows no sign yet of surrender. The Moore Library will bring together books and journals currently held at four locations and as part of Cambridge University Library will house relevant material received under Legal Deposit legislation. More than 7,000 metres of custom designed shelving on four floors will provide an initial capacity for around 156,000 volumes.

A second requirement is the provision of a cluster of 70 reader workstations and power and data connections to remaining reader places so that the library can respond quickly to future changes in the balance of print/electronic library use. The library will be active in the administration of networked electronic services and also a key training and support centre.

Photograph courtesey of Eaden Lilley

Probably the most important factor in the success of any library is that the building offers pleasing yet functional spaces to visit, work and interact in. A library should be living, social space and the Moore Library has been designed for people and not just as a book store. The circular design of the upper floors provides natural circulation areas around the central lift shaft and twin stair wells with the shelves fanning out to seating around the perimeter. Current periodicals will be displayed on the first floor while on the ground floor recent book acquisitions will be displayed in a casual seating area opposite a single service desk that fronts the library staff work zone.

Since the University Librarian and I first met with Edward Cullinan and his team in early 1997 to discuss the project progress has been rapid, the experience immensely rewarding. Next year will confirm the arrival of one of the world's great science libraries.

The Faulkes Institute for Geometry

Professor W. B. Raymond Lickorish, Head of Department, Dept of Pure Mathematics & Mathematical Statistics

It was with profound gratitude that the mathematical community in Cambridge learnt of the recent donation from Dr Dill Faulkes. Several donations of breathtaking generosity have been made to the Centre for Mathematical Sciences, but this one means that finance of the main building programme is just about complete and we can contemplate the time when the whole project will be finished. Indeed, the whole northern side may be finished by the end of 2001. Dr Faulkes' imaginative request that Pavilion E become an Institute for Geometry has given real encouragement to those whose primary interest is in mathematics as an intellectual adventure.

Geometry comes from long ago. From about 1100 BC, at the start of their aptly named Geometric Era, the Greeks were investigating geometric designs and patterns and it was the Greeks who, some six centuries later, introduced western civilisation to the theory of lines and angles, circles and conic sections, congruent triangles and cyclic quadrilaterals. Most of us can recall our own 'long ago' when we learnt, albeit imperfectly, the basic procedures of Euclid's philosophy. Happenings concerned with lines, angles and circles, instances of which could readily be checked with the ruler and compass of the trusty geometry set, were formally proclaimed as 'theorems' or 'lemmas' (or maybe 'lemmata'). These statements were meticulously proved, starting from postulates and previously established results. In this way we gained, not simply an acquaintance with the space in which we live, but a fundamental understanding of what mathematics is and how mathematical results must be substantiated. We might even have learnt that, in any worthwhile study, there is a *pons asinorum* (Euclid's fifth proposition) that must be crossed to obtain understanding!

This ancient spirit of Euclidean geometry, whereby a result is incontrovertibly proved to the profound satisfaction of its author, is still the essence of the whole of pure mathematics today (though now use of the triumphal 'Q.E.D.' is a little out of fashion). In substance, geometry is concerned with space, or spaces, of every conceivable sort. Spatial thinking dominates a great deal of mathematical research; a new theory often begins with an intuitive visual idea about points or objects in space that is then later transformed into a rigorous logical argument. However, within the actual repertoire of modern mathematics, the subject of geometry itself still occupies a central key position. It is on a more or less equal footing with algebra and analysis, subjects with which it often blends. Modern geometry includes a variety of different topics which will be pursued by different but overlapping research groups at the Institute for Geometry. A deep appreciation for Euclidean geometry is necessary for modern geometers, necessary but by no means sufficient.

Abstract group theory is the study of all possible forms of symmetry. Symmetry being a most fundamental and attractive aspect of geometry, it is proper that the study of group theory should be located in the Institute for Geometry. Conversely, the basic idea of the famous Erlanger programme of Felix Klein was that geometry *is* the study of spaces acted upon by groups of symmetries. In Euclidean geometry the symmetries are the translations, rotations and reflections; for these are the distance-preserving changes that, for example, move a triangle to a congruent triangle. However many other sorts of movements, like the expansions and contractions which preserve angles but not distances, or line-preserving projections from one plane to another, are also candidates. Such ideas can be considered in any number of dimensions and in vast generality. More importantly they can be *localised*, resulting in the study of *manifolds*. These are spaces that, in all small enough areas, are equivalent to Euclidean space but in toto are quite individualistic. The sphere, the torus or the Mobius band are visualisable examples. 'Equivalence' is always a loaded word in mathematics. Here, variations of its meaning lead to the objects of study used in differential topology, algebraic geometry, Lie group theory, differential geometry (including the hyperbolic geometry of Lobachewsky which, by defying Euclid's fifth postulate, so disturbed thinkers in the nineteenth century), differential equations, and the spaces of Einstein's relativity theories.

These topics of modern geometry, and many more, will be studied in the Faulkes Institute. It will house the heirs to what is a long tradition of geometry in Cambridge; the Lowndean Professorship of Astronomy and Geometry was endowed in 1749. Apart from offices, the building will contain a fine lecture room, a necessary committee room and special display cases for the faculty's classic collection of the plaster models of geometric objects, a few of which have been wonderfully but expensively cleaned and restored. It is intended that having the Institute in Cambridge will emphasise to all that the whole field of geometry is still of enormous intellectual, practical and pedagogical importance in modern times.

The Story of M

Professor P.K. Townsend, Department of Applied Mathematics & Theoretical Physics

They were learning to draw, the Doormouse went on, ... *and they drew all manner of things — everything that begins with M. Why with an M*? said Alice. *Why not*? said the March Hare.

Since 1995, all manner of things have been drawn in to a new physical theory, still under construction, known as 'M-theory'. M-theory takes over where superstring theory left off, somewhere on the long road towards a unification of the quantum theories of particle physics with Einstein's theory of gravity, General Relativity. And why 'M'? Well, why not? The ambiguity in the name reflects the provisional nature of our understanding of the theory. This, at any rate, is the standard prevarication. Protest that it sounds a little too much like the March Hare and you may hear it whispered that the Mad Hatter's view is that M stands for 'Membrane'.

Superstring theory posits that the particles like quarks and neutrinos that we see in particle physics experiments (in addition to the ones we have yet to see, like gravitons) are all vibrational modes of an elementary one-dimensional object; a 'superstring'. This theory resolved many of the problems encountered in previous attempts to fuse quantum mechanics with General Relativity, but still left unanswered questions. To begin with, there are five different superstring theories, four more than needed for a unified theory. More seriously, none of them is capable of explaining the properties of, say, a Black Hole. So the search for a unified and complete theory was by no means over. Around 1986, a small group of theorists, with DAMTP taking the lead, began to wonder whether the 'elementary objects' of such a theory might not be *two-dimensional* 'supermembranes' rather than superstrings.

A key feature of superstring theory is its need of a *ten-dimensional* spacetime. This may sound like an adventure suitable for Alice, but it is really a blessing in disguise because the process of 'compactification' by which superfluous dimensions are rendered effectively invisible also allows the emergence of many realistic features that any complete theory must incorporate. This motivation for postulating additional, but hidden, dimensions of space goes back to the 1920s and was taken up periodically by Einstein himself, without much success however. Superstring theory does better partly because the additional dimensions are not simply postulated but instead *required* for internal consistency.

Of course, one can't really know what consistency requires until one has a consistent theory and superstring theory just fails to qualify. The theory is really a prescription for computing the outcomes of physical processes as an infinite sum of terms. For some purposes the first few terms may suffice, and within the realm of such approximations the theory is consistent. But if one wants to apply it to study black holes, for example, then the infinite series of terms must be summed. Unfortunately, the series diverges so the answer is nonsense. This is typical of series approximations and not in itself a disaster, but it does mean that the underlying theory approximated by superstring theory need not actually be a theory of strings. Nor is spacetime necessarily ten-dimensional. Instead, the correct inference, given the premises of superstring theory, is that spacetime is *at least* ten-dimensional.

This is just as well for membranes because supermembrane theory requires *eleven* dimensions. Actually, this was the origin of the idea that strings might need to be replaced by membranes: the eleven dimensions came first, back in 1978, when 'eleven-dimensional supergravity' emerged as a leading candidate for a unified theory. Internal difficulties led to its decline, and it was set aside after the 1984 'superstring revolution'. Supermembrane theory was an attempt to revive it. Strings were to be explained as membranes wrapped on the eleventh dimension. But when the implications of quantum mechanics for supermembranes were finally worked out, in 1988, the result was a theory that was, apparently, too strange to be relevant.

Meanwhile, attempts beginning in 1994 to get to grips with the problem of unifying the five superstring theories began to focus on the idea that they were 'dual' realizations of a single theory. To get this to work, account had to be taken of a whole menagerie of 'branes', including particles, strings, membranes and many others, such as 'fivebranes'. Crucially, unification could be achieved only by postulating an eleven dimensional theory, from which eleven-dimensional supergravity and all five superstring theories would emerge as limiting cases. This theory was soon dubbed 'M-theory' but what was it? Supermembranes clearly played a role but so too, each in its own way, did all the other 'branes'. What was lacking was a definition of the theory in terms of basic constituents. Ironically, initial progress came from a return to particles and a so-called 'M(atrix) model', but it was soon realized that this was identical to the earlier quantum supermembrane theory, the only difference being one of interpretation. In superstring theory the scattering of n particles is described by replacing the particles by n strings. This cannot work for supermembranes. Instead, the same process is described in terms of deformations of a *single* supermembrane!

Image courtesey of Athens Antique Prints

However, just as the search didn't end with strings, neither does it end with membranes! Although supermembrane theory provides us with a consistent theory of quantum gravity in *eleven* dimensions, it turns out to be much harder than expected to 'compactify' it. As one reduces the dimensionality it becomes necessary to consider other branes, and there is evidence that the 'fivebranes' dominate below eight dimensions. So we still don't really know what M-theory is. Perhaps it is a 'brane democracy' of some sort. Current attempts to understand the implications of M-theory involve the idea that our world is a 'braneworld' floating in a higher dimensional spacetime. Another adventure for Alice? To those involved in the race to find a truly unified theory of particle physics and General relativity, the Red Queen's advice to Alice often seems prophetic:

Here, you see, it takes all the running you can do to keep in the same place. If you want to get somewhere else you'll have to run at least twice as fast as that.

Professor Hawking, Marilyn Monroe and Me

by Karen Sime, Personal Assistant to Professor Stephen Hawking, Department of Applied Mathematics and Theoretical Physics

Having worked in the same office at the Old Press Site in Silver Street for more than 27 years, Professor Hawking's move to the new CMS building in January 2000 was a major change of environment for him. Room G2 was certainly full pitch & moment, *seemingly* cosy and stacked from floor to ceiling with comfortingly familiar items from his life's work. But, to be perfectly honest, it was *so* cramped that there wasn't enough room to swing Schroedinger's cat! In Winter, there were the spine-chilling draughts from the windows that didn't *quite* close and, although I am of robust and Rubenesque proportions myself, even *I* had been known to occasionally sport a scarf and padded jacket when going through the mail with Professor Hawking. In summer, we either choked on exhaust fumes from the car park and outside traffic, or boiled like potatoes with the windows shut. The CMS *had* to be an improvement on *this*!

Photograph courtesy of Mike Finn-Kelcey

Socialising during "afternoon tea" was a very enjoyable and important part of Professor Hawking's professional life at Silver Street, and I *did* have my doubts as to whether the CMS would have the same ambiance as the *old* DAMTP. Of much more concern, however, was the prospect of being considerably further from the town centre, thereby bringing to an untimely end our frequent sorties to the local patisserie for Professor Hawking's afternoon cakes. The CMS *does* have a brand, spanking new canteen, but "crawling to the boss" with a University Centre sandwich just doesn't have the same impact as a fresh cream meringue from Fitzbillie's.

Nevertheless, the entire General Relativity Department entered into the spirit of this big, new adventure, and we crammed our antiquated, bulging files into the smart, minimalist-style filing cabinets of the new building. Professor Hawking also brought along his photo collection of Marilyn Monroe. Wherever you looked in his office, Marilyn was *there*, serving as a constant reminder of how utterly ordinary the rest of us were. Who'd have thought that in years to come she would have found new fame as a *fly-on-the-wall* in the office of one of the world's greatest physicists! I thought we might have made a fresh start and left her to gather dust back at Silver Street but, alas, no such luck.

At first, there were a few hiccups at CMS. The automatic blinds, for example, whizzed up and down of their own accord at the most *in*opportune moments. On one particularly memorable occasion, I decided to change into my suit at my desk having just returned from the gym. At that time, the builders were still working directly outside my window in huge JCBs almost at first floor level. I closed the blinds but, to my chagrin, they decided to roll themselves up again whilst I was in mid-*déshabille*, leaving little to the imagination and causing much amusement to the site staff. On hearing this news, Professors Hawking and Turok could barely conceal their mirth. I, on the other hand, immediately took to skulking around the building in a raincoat and dark glasses for some considerable time afterwards.

Now that we've been here for almost nine months, it is evident that the CMS scores many *plus* points for Professor Hawking. His new office has a marvellous view which, I feel, is wonderfully therapeutic, and the building itself allows him much freedom to come and go as he pleases. The modern and spacious *feel* of the place is very American/Scandinavian, thus complementing the accent of his Speech Synthesizer rather well! There is plenty of ventilation and the windows close securely, keeping us snug and warm in winter without the need for oilskins, snow boots and thermal underwear. The staff restaurant offers Professor Hawking an excellent choice of meals at lunchtime, and he can still enjoy "afternoon tea" with his colleagues and students in bright, modern surroundings.

I think there will always be a little place in Professor Hawking's heart for G2 at Silver Street, but there is certainly never a dull moment in B1.07 where the omnipresent Miss Monroe continues to keep a watchful eye upon the Lucasian Professor of Mathematics.