Asymptopia

CENTRE FOR MATHEMATICAL SCIENCES NEWSLETTER

UNIVERSITY OF CAMBRIDGE

September 2000

It's Officially Open

Patrick Hawke-Smith, CUDO

On Thursday 13 July the new Centre for Mathematical Sciences was inaugurated by HRH, The Duke of Edinburgh, Chancellor of the University. The Duke was shown the progress of the building works, touring the two completed pavilions and crossing the grass roof of the central core. He also met some of the mathematicians and physicists already working at the Centre, including Professor Stephen Hawking.

The occasion was brought to wider public attention by the Duke's favourable comparison of the new Centre with the Millennium Dome. His comments, which appeared to provoke little controversy among those present, were made in the context of a brief speech preparatory to unveiling a commemorative plaque. The Duke then finished his visit by meeting some of the Centre's leading benefactors.

After the Chancellor's departure for the University Honorary Degree ceremony, guests continued to enjoy the hospitality of the Centre. Two special lectures given in the Wolfson Room provided an entertaining insight into the work of each of the Mathematics departments: Professor Timothy Gowers of DPMMS spoke on *Why it is Hard to Think about Prime Numbers*, and Professor Neil Turok of DAMTP on *The Bottom Line in Cosmology*.

Guests then had the opportunity to view the new buildings for themselves, with architects, project managers and mathematicians on hand to explain both the design and how the Centre was benefiting scholarship in practice. The response was overwhelmingly positive. Mr Peter Gershon, Chairman of the External Fundraising Committee, was especially pleased "to hear so many of the staff talking so enthusiastically about the new facilities."

The inauguration marks the culmination of the first phase of the construction of the Centre. The second and final phase is due to be completed in late 2002, although there is much work, including quite a bit of fundraising, to be done before then. Inspired by the inauguration, however, we look forward to an equally splendid occasion at the official opening of the Centre as a whole.

What is the Millennium Mathematics Project?

Julia Hawkins, Millennium Mathematics Project

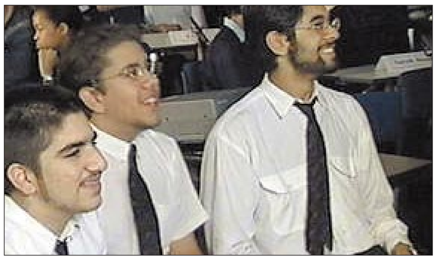
The Millennium Mathematics Project (MMP) is a new long-term national initiative, based in Cambridge, which aims to improve the understanding and enjoyment of mathematics among school pupils and the general public. We want to dispel the myth that maths is a 'difficult', boring and abstruse subject, and to help people of all ages and abilities understand the vital contribution of mathematics and its applications to science, commerce and everyday life.

The MMP has been built on the success of three existing projects: NRICH and *Plus* (formerly PASS-Maths) - both web-based programmes providing resources for students and schools - and STIMULUS, a peer-assisted learning scheme. Since our launch in July 1999 we've continued to develop new programmes, ranging from a video-conferencing project to support mathematics in disadvantaged schools to a planned series of workshops on the interface between maths and art. Moreover, since coming beneath the umbrella of the MMP last year both NRICH and Plus have continued to grow from strength to strength.

NRICH is an **online maths club** for children from 5 to 18, with dedicated sections for both primary and secondary education, providing free teaching resources and mathematical challenges to be tackled by pupils outside school mathematics classes. The material is designed to take children beyond the confines of school syllabuses, to help them enjoy problem solving, and to appreciate the significance and range of applications of mathematics. Every month we publish a new issue containing mathematics problems, solutions sent in by children, news, articles, games and interactivities.

Mathematical questions from teachers and children are answered by the **AskNRICH Answering Service** staffed by university students. There is a choice of either a one-to-one service where questions are answered on an individual basis and email mentoring of able students is possible, or of posting queries on a bulletin board open to all readers to participate in more general discussion. This service is extremely popular and attracts both teachers (for whom there is a special

The teachers' chat room on the NRICH website, part of the MMP, is very popular



Pupils during the first MOTIVATE video-conferencing session

private chat area) and pupils, while the topics covered overlap with physics as well as maths.

The NRICH website now has around 40,000 users in over 80 countries and receives on average 10,000 hits per day.

Plus is a complementary **online magazine** for older pupils and the general public. It publishes articles explaining the diverse applications of mathematics within physics, chemistry, biology, engineering, and economics. It provides stories about new developments in mathematics and mathematical sciences and interviews with mathematicians. It also features information about degree courses, the history of mathematics and science, mathematical biographies, and links to other mathematics websites and resources. *Plus* shows readers something of what professional mathematicians do and explores the wide range of careers open to mathematics graduates.

Under the **STIMULUS** programme, university students visit local schools to help with mathematics and science teaching. The presence in schools of these enthusiastic and able young people provides important role models for pupils and helpful careers advice in addition to mathematical tutoring.

Perhaps the most exciting new development last year was the launch of **MOTIVATE**, a unique and exciting new pilot **video-conferencing** scheme which began in the Autumn term of 1999. The pilot project has been chosen for funding by the National Endowment for Science, Technology and the Arts as one of its pioneer projects. MOTIVATE aims to bring schoolchildren from disadvantaged areas into direct contact with world-class research mathematical scientists to enhance the teaching of mathematics within schools and broaden the horizons of expectation for gifted pupils. MOTIVATE has attracted considerable press interest, including coverage on BBC News Online and the Daily Telegraph.

We are looking forward to the next year and have many plans to continue enriching maths education in schools and to improve the public perception of mathematics. A teachers' conference took place in Cambridge in July, bringing together research mathematicians, policy makers and schoolteachers to explore practical ways in which ICT can help add enthusiasm to the maths classroom, while the video-conferencing project was extended internationally with a link to schools in South Africa. Other exciting projects are in development, including a web-based **interactive multilingual mathematics dictionary,** frequent **public talks** and collaborative workshops and an exhibition, to be held in 2001, on the interplay between **maths and the visual arts**.

For more information please visit the MMP website at http://www.mmp.maths.org.uk or our individual programme websites at http://www.nrich.maths.org, http://www.plus.maths.org and http://www.stimulus.maths.org.

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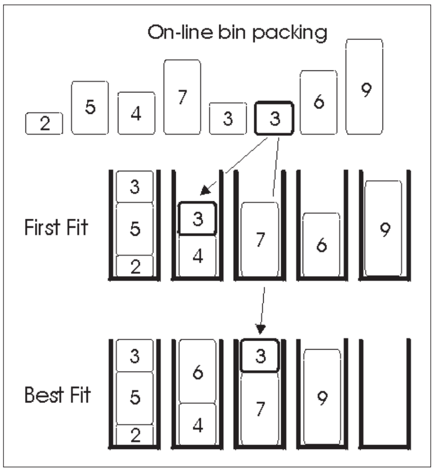
On-Line Bin Packing

Richard Weber, Churchill Professor of Mathematics for Operational Research,
Statistical Laboratory, Department of Pure Mathematics and Mathematical Statistics



Suppose you have taken 1000 photographs with your digital camera and these are stored in files of different sizes. You want to copy the files onto a minimum number of 1.44Mb diskettes. If the total size of the files is exactly 144Mb you will need at least 100 diskettes. However, because you may be unable to fill each diskette completely you will need more than 100 diskettes. If 110 diskettes are required, then there is ‘wasted space’ of 14.4Mb (that is, 10 times 1.44).

Bin packing problems, like the one above, occur in the design efficient methods for data storage, allocation of computer memory, and packing data into the packets that are transmitted in the Internet. The manufacturer of Levi’s jeans faces a two-dimensional bin packing problem when he seeks to cut pieces for pockets, legs, etc., from the least amount of denim cloth.

Bin packing problems lie in a class of problems called NP-hard. This class includes scheduling problems, graph-coloring problems, and also the famous ‘travelling salesman problem’, in which the challenge is to find the shortest path by which a salesman can visit n cities and return to his start. An important application is to plan the order in which a robot can most quickly insert n components onto a printed circuit board. A polynomial-time algorithm for this problem would find the optimal order in a time which grows no more quickly that n to a power, say n-cubed. NP-hard problems are equally difficult, in the sense that if one could find a polynomial-time algorithm to solve just one of them, then this could be done for all. It is an open conjecture that no polynomial time algorithm exists for NP-hard problems and that computation time must grow exponentially in n, e.g., as 2 to the power n. This puts the optimal solution of many practical problems out of the reach of even the most powerful computers.

Our research has focused on designing near-optimal algorithms for bin packing when the packing is to be done on-line. That is, n items are presented for packing one at a time and must be packed into bins with no subsequent re-packing permitted. Bins are of integer size k. Item sizes are drawn randomly and uniformly from the set of integer 1 to j, where j is less than k-1. In collaboration with others, we have proved that under an optimal on-line algorithm the expected wasted space is bounded, i.e., it need not grow with n. However, if $j=k-1$ then the expected wasted space must grow as the square root of n. Amongst other things, these results require proof of the following non-trivial theorem. Suppose r, j, m and k are any integers such that $r(1+2+\cdots+j)=mk$ and $j<k$. Show that it is always possible to take a collection of rj items, consisting of r items of each of the sizes 1 to j, and pack these into m bins of size k, with no wasted space.

Two possible on-line algorithms are First Fit (FF) and Best Fit (BF). FF takes each successive item and places in the first bin in which it fits. BF takes each successive item and places it in the partially full bin that it most nearly fills, or into an empty bin if it fits in no partially full bin. In the accompanying illustration the bins are of size 10. The second item size 3 is packed differently by FF and BF. In the illustration BF is optimal, and in practice BF usually works very well. However, when $j=8$ and $k=11$ the expected waste grows very slowly, but linearly, in n (whereas we have proved that under an optimal algorithm the expected wasted space is bounded). The proof that BF does poorly in this case took several years to piece together and led to the development of some new mathematics. It involved new results about the recurrence properties of multi-dimensional Markov chains. Like the proof of the Four Colour Theorem, the proof has a step that relies on a computer, which is needed in this case to check a very large number of inequalities.

Recently we have invented a new algorithm, SS. This algorithm packs items so as to minimize at each step the sum of the squares of the numbers of partially full bins with remaining gaps of sizes 1 to k-1. Remarkably, we have proved that for all distributions of item size (not only the uniform distribution) the expected wasted space under SS behaves in the same way as it would under an optimal algorithm. It can be bounded, or grow linearly or as the square root of n.

You may be surprised to learn that something as prosaic as the machinery used to pack chicken pieces into multi-packs for sale in supermarkets uses sophisticated on-line algorithms. The requirement might be that each pack contains 6 drumsticks and that the total weight is at least 400 grams. The machinery aims to assemble pieces into packs that weigh on average as little as possible over 400g. We are applying insights from our on-line bin packing research to design better algorithms for this machinery.

Swimming micro-organisms

T J Pedley, G I Taylor Professor of Fluid Mechanics, Department of Applied Mathematics and Theoretical Physics

Multitudes of micro-organisms exist in almost every conceivable aqueous environment on earth and have been estimated to form a major part (more than 50%) of the world’s biomass. Most individuals live in the oceans, because that is where most of the water is. A large fraction of them constitute the phytoplankton, the light-converting bottom link of the food chain, but many similar species also live in large or small bodies of fresh water. These micro-organisms, mostly algae, dinoflagellates, diatoms, etc, are of considerable scientific interest for various reasons: they are thought to absorb more carbon dioxide, *in toto*, than terrestrial plants, so understanding their biology and correctly estimating their numbers is vital to forecasts of the greenhouse effect; patchiness in their populations can have significant effects

Swimming micro-organisms *continued*

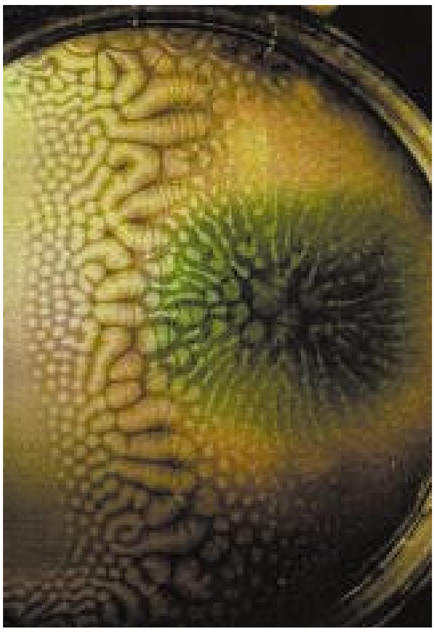
on populations of higher organisms, either beneficial (e.g. stimulating the schooling of krill, exploited as food by even larger animals such as whales or men) or deleterious (e.g. red tides, which poison coastal seafood industries); some of them are harvested for their food value or for the chemicals they produce (*Dunaliella salina*, for example, is used as a source of beta-carotene, a popular red food colouring – it is more 'green' for food-colouring to be manufactured by algae than by chemists); others, which swim, are used as a means of bio-assay of water quality (if they stop swimming then the concentration of something harmful is too large – they can be thought of as small green canaries). Moreover, every photosynthetic micro-organism is likely to be host to several even smaller creatures, the bacteria.

The Biological Fluid Dynamics Group in the Department of Applied Mathematics and Theoretical Physics (DAMTP) is best known for its work on physiological flows: blood flow in arteries and veins, airflow in the lungs, surface tension effects in the thin liquid lining of the pulmonary airways, the pumping of urine from the kidneys to the bladder, and a host of other internal transport processes. However, the other, external, "arm" of biological fluid dynamics concerns the interaction of living organisms with their fluid environment. The most obvious problems arise in the mechanics of swimming (of fish and other aquatic organisms, including algae and bacteria) or flying (of birds, bats and insects). And in recent years the group has been studying the collective behaviour of populations of swimming micro-organisms.

There have been two motivations for this work. The first comes from the computational models some oceanographers use to simulate the oceanic ecosystem. The rate of change of population for each species in the model (an alga, say) is given by a mathematical equation in which the various terms represent the birth-rate, natural death rate, growth rate through uptake of nutrient (itself dependent on the light intensity, which is influenced by the local population density) and rate of predation by the next higher level in the food chain (often copepods, which are small crustacea). Then there will be an equation for the growth rate of copepods, involving their predation by small fish, and so on. With several different species in the model, the number of interacting equations rapidly becomes large, requiring substantial computational effort. And the results, in the few cases that can be tested, rarely bear much resemblance to observation. The reason lies, at least in part, in the fact that fundamental information on the terms in the equations is lacking. For example, the rate at which copepods eat algae depends on how frequently they encounter them, and that is crucially dependent on the state of turbulence in the water, which is quite different in the open ocean from anything that can be produced in a tank in the laboratory, so the encounter rate cannot be directly measured. There is in DAMTP a project to estimate the encounter rate between randomly swimming micro-organisms in turbulent flow, an endeavour that involves a subtle interplay between probability theory and fluid dynamics. Another example is the rate of nutrient uptake by the algae. Most models assume a rate similar to that for inert, sedimenting particles, but an organism's swimming motions stir up the fluid around it, enhancing the uptake. We are engaged in a fundamental theoretical study to assess this effect, which can be substantial.

The other motivation for the work arose out of some fascinating laboratory experiments by Professor John Kessler of the University of Arizona. He found that fairly dense populations of certain swimming micro-organisms would spontaneously form regular patterns. An example in a population of the bacterium *Bacillus subtilis* is shown in the figure. The basic mechanism for the pattern-formation is quite simple: the cells swim upwards, on average, and are heavier than water, so in the shallow dish the mass density near the top surface is greater than that at the bottom. Such a stratification is unstable and leads to a convective motion similar to that observed in a layer of fluid heated from below (the pattern-forming process is therefore called "bioconvection"). In an attempt to provide a mathematical model of bioconvection, which we hope will enable us to predict the size and shape of the patterns, we have been confronted with a number of difficulties. For instance, the reason the bacteria swim upwards is in fact that they swim up gradients of oxygen concentration. They themselves create the gradients by consuming the oxygen so that its concentration falls everywhere except at the upper surface which is in contact with the atmosphere. Thus there is an intricate, nonlinear coupling between the cell and oxygen concentrations which must be incorporated into the model. Another complication, which we are only just beginning to tackle, is the fact that, once the convective motions begin, they not only tend to distort the cell and oxygen concentration distributions, but they also strongly influence the random swimming trajectories of the bacteria themselves. Thus it is no longer clear what their average swimming direction will be. The complications involved in describing such small-scale, low-speed collective behaviour only serve to underline how daunting is the task of simulating ocean ecology.

Fortunately, we have succeeded in obtaining some financial support for this work, both from the Natural Environment Research Council and, recently, from the European Commission. There are many theoretical problems to be solved in this area, and DAMTP intends to remain at the forefront of the effort to solve them.



Bioconvection patterns observed from above in a shallow dish, slightly tilted from the horizontal, containing a suspension of bacterial cells (Bacillus subtilis). Dish diameter, 10cm; maximum depth, 7mm (approx); cell number density, 10^8 cells per ml (approx). The pale segment to the left is dry; there are no patterns in the shallowest portion of the suspension; hexagonal patterns are seen when the depth slightly exceeds a critical value, giving way to more complex, segmented patterns at greater depth. The green patch contains dye, a blob of which was dropped into the mixture to see how much the convective motions enhance lateral mixing. Photograph courtesy of J O Kessler, University of Arizona.